BIRTHDAY PROBLEM

MELIKAMP

The problem can be stated as follows. In a group of n people whose birth dates are distributed randomly, independently and uniformly, what is the probability of the event B_n that at least 2 people will have the same birth date? To approach this question, we notice that if A_n is the event "no 2 people have the same birth date", then A_n and B_n are complimentary events and $P(B_n) = 1 - P(A_n)$. For simplicity, we will assume that there are always 365 days in a year.

By the classical approach to probability, $P(A_n)$ is the ratio of the number of favorable outcomes to the total number of outcomes. For a fixed number n of people in our group, the total number of ways to assign birthdays is 365^n. On the other hand, the number of ways to assign birthdays without collision is $365 \cdot 364 \cdot \ldots \cdot (365 - n + 1)$ (by the basic principle of counting). It follows that

$$P(A_n) = \frac{365 \cdot 364 \cdot \ldots \cdot (365 - n + 1)}{365^n} = \frac{365!}{365^n (365 - n)!},$$

and therefore

$$P(B_n) = 1 - \frac{365!}{365^n (365 - n)!}.$$

Following is the table with probabilities of B_n for a few selected n.

n	1	2	3	4	5	10
$P(B_n)$, %	0.00	0.27	0.82	1.64	2.71	11.69

n	15	20	30	40	50	100
$P(B_n)$, %	25.29	41.14	70.63	89.12	97.04	99.99

To summarize, the chances are pretty good (4 : 6) that in a group as small as 20 people there will be a collision.

For the reference, here is some code in Lisp to compute $P(B_n)$. It was tested in Emacs and Common Lisp.

```
(defun birthday (n)
  (if (eq n 1) 1 (* (/ (+ 365.0 (- n) 1) 365.0) (birthday (- n 1)))))
(defun nbday (n)
  (- 1 (birthday n)))
```

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