HW SOLUTIONS

MELIKAMP

1. Problems due by July 13

All drawings are omitted. All equalities in computations should be assumed to be approximate, even though = is used.

Problem (2.6.1).

Solution. $n = 6$, $(85, 92, 103, 108, 128, 156)$.

(a)
$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = 112.$$
(b)
$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}} = 26.15.$$
(c)
$$Q_2 = \frac{103 + 108}{2} = 105.5.$$
(d) Using $\lceil \frac{n}{4} \rceil$ to find the index of the first quartile, $Q_1 = 92$, $Q_3 = 128$.

□

Problem (2.6.5).

Solution. $n = 8$, $(46, 77, 83, 84, 85, 90, 91, 94)$.

(a) $\overline{X} = 81.25$.
(b) $Q_2 = 84.5$.
(c) $s^2 = 231.36$.
(d) $s = 15.21$.
(e) $Q_1 = 77$, $Q_3 = 91$.

□

Problem (2.6.9).

Solution. Here n is the sum of frequencies, $n = 52$.

(a) $Q_2 = 128$.
(b) The index is $\lceil 52/4 \rceil = 13$, $Q_1 = 120$, $Q_3 = 136$.

□

Problem (2.6.18).

Date: July 20, 2009.

Solution. $n = 25$.

65	72	76	77	78
79	**81**	82	83	83
84	84	**85**	85	86
88	89	89	**92**	93
94	94	97	97	99

(c) The median is the 13-th data point, $Q_2 = 85$.
(d) $\lceil 25/4 \rceil = 7$, $Q_1 = 81$, $Q_3 = 92$.
(e) The sample range is $\max - \min = 99 - 65 = 34$.

□

Problem (3.9.4).

Solution. Completed table:

Gender	Private	Medicaid	None	
Female	250	452	208	910
Male	128	680	157	965
	378	1132	365	1875

(a) $P(\text{no insurance}) = 365/1875 = 0.195$.
(b) $P(\text{female} \cap \text{no insurance}) = 208/1875 = 0.111$.
(c) $P(\text{female} \mid \text{no insurance}) = 208/365 = 0.57$.
(d) $P(\text{female}) = 910/1875 = 0.485 \neq P(\text{female} \mid \text{no insurance})$, so the events are dependent.

□

Problem (3.9.5).

Solution. $P(T|D) = 0.8$, $P(D) = 0.3$, $P(T') = 0.76$. One can assemble the following table by having putting 1 for total (since $P(\Omega) = 1$).

	D	D'	
T			0.24
T'			0.76
	0.3	0.7	1

Also, $P(T|D) = 0.8 = \frac{P(T \cap D)}{P(D)}$, so

$$P(T \cap D) = 0.8 \cdot P(D) = 0.8 \cdot 0.3 = 0.24,$$

and the rest of the table can now be completed:

	D	D'	
T	0.24	0	0.24
T'	0.06	0.7	0.76
	0.3	0.7	1

(a) $P(D \cap T) = 0.24$.
(b) $P(T) = 0.24$.

(c) $P(T' \cap D) \neq 0$, so these events are not mutually exclusive.
(d) $P(T)P(D) = 0.24 \cdot 0.3 = 0.072$, while $P(T \cap D) = 0.24$, so these events are dependent.

□

Problem (3.9.13).

Solution. The answers follow directly from the completed table, not shown.

(a)

$$\begin{aligned} P(\text{eligible}) &= P(\text{eligible-enrolled} \cup \text{eligible-refused}) \\ &= (150+55)/300 = 0.683. \end{aligned}$$

(b) $P(\text{ineligible}) = 95/300 = 0.317$.
(c) $P(\text{enrolled} \mid \text{male}) = 80/135 = 0.593$.
(d) $P(\text{female} \mid \text{ineligible}) = 60/95 = 0.631$.
(e) $P(\text{eligible and male}) = (80+20)/300 = \frac{1}{3}$, while

$$\begin{aligned} P(\text{eligible})P(\text{male}) &= 0.683 \cdot (135/300) \\ &= 0.683 \cdot 0.45 = 0.307 \neq \frac{1}{3}, \end{aligned}$$

so these events are dependent.

□

Problem (3.9.14).

Solution. This is a binomial experiment with $n = 12$ and $p = 0.8$.

(a) $P(X = 10) = 0.2835$.
(b) If $p = 0.82$, then

$$\begin{aligned} P(X=10) &= {}_{12}C_{10} \cdot 0.82^{10} \cdot 0.18^2 \\ &= \frac{12!}{2!10!} \cdot 0.82^{10} \cdot 0.18^2 \\ &= \frac{11 \cdot 12}{2} \cdot 0.82^{10} \cdot 0.18^2 \\ &= 0.294. \end{aligned}$$

(c) If $p = 0.8$ again, then the expected value of X, $EX = pn = 0.8 \cdot 12 = 9.6$.

□

Problem (3.9.18).

Solution. This is a binomial experiment with $p = 0.4$.

(a) If $n = 10$, then

$$\begin{aligned} P(X \geq 4) &= 1 - P(X \leq 3) \\ &= 1 - (P(X=0) + P(X=1) + P(X=2) + P(X=3)) \\ &= 1 - (0.0060 + 0.0403 + 0.1209 + 0.2150) \\ &= 0.6178. \end{aligned}$$

(use the table). Here and below we use the probability axiom for complementary events in order to reduce the number of terms in our calculations.

(b) With $n = 10$,

$$\begin{aligned} P(X \le 8) &= 1 - P(X > 8) \\ &= 1 - (P(X = 9) + P(X = 10)) \\ &= 0.9983. \end{aligned}$$

(c) $EX = 5250 \cdot 0.4 = 2100$.

□

Problem (3.9.25).

Solution. $X \sim N(24, 2.5^2)$.

(a)

$$\begin{aligned} P(X > 22) &= 1 - P(X \le 22) \\ &= 1 - P(Z \le \frac{22 - 24}{2.5}) \\ &= 1 - P(Z \le -0.8) \\ &= 1 - 0.2119 \\ &= 0.7881. \end{aligned}$$

(b)

$$\begin{aligned} P(15 < X < 25) &= P(X < 25) - P(X < 15) \\ &= P(Z < 0.4) - P(Z < -3.6) \\ &= 0.6554 - 0 = 0.6554. \end{aligned}$$

(c) Look up 0.05 in the middle of the table (the 5-th percentile) in order to find that

$$P(-1.64 < Z < 1.64) = 0.9.$$

Then the 5-th percentile for X is given by

$$-1.64 \cdot 2.5 + 24 = 19.9,$$

and the 95-th percentile is

$$1.64 \cdot 2.5 + 24 = 28.1$$

□

Problem (3.9.26).

Solution. $X \sim N(500, 100^2)$.

(a)

$$\begin{aligned} P(580 < X < 620) &= P\left(\frac{580 - 500}{100} < Z < \frac{620 - 500}{100}\right) \\ &= P(0.8 < Z < 1.2) \\ &= 0.8849 - 0.7881 = 0.0968. \end{aligned}$$

(b) $P(400 < X < 800) = 0.84$.

(c) The 90-th percentile for Z is 1.28 (look up 0.9 in the middle of the Z table), so 90-th percentile for X is

$$1.28 \cdot 100 + 500 = 628.$$

□