HW SOLUTIONS

MELIKAMP

1. Problems due by July 23

All drawings are omitted. All equalities in computations should be assumed to be approximate, even though = is used.

Problem (4.5.4).

Solution. It is given that $\sigma = 6.3$.

(a) Given $n = 40$ we can write

$$\begin{aligned} P(\overline{X} < \mu + 1) &= P\left(Z < \frac{\mu + 1 - \mu}{\sigma/\sqrt{n}}\right) \\ &= P\left(Z < \frac{1}{6.3/\sqrt{40}}\right) \\ &= P(Z < 1) \\ &= 0.8413. \end{aligned}$$

(b) With $n = 100$ and the same σ we can write

$$\begin{aligned} P(\overline{X} < \mu + 1) &= P\left(Z < \frac{\mu + 1 - \mu}{\sigma/\sqrt{n}}\right) \\ &= P\left(Z < \frac{1}{6.3/\sqrt{100}}\right) \\ &= P(Z < 1.59) \\ &= 0.9441. \end{aligned}$$

□

Problem (4.5.8).

Solution. It is given that $\mu = 182$ and $\sigma = 14.7$.

(a) Here $n = 20$, and

$$\begin{aligned} P(180 < \overline{X} < 185) &= P(\overline{X} < 185) - P(\overline{X} < 180) \\ &= P\left(Z < \frac{185 - 182}{14.7/\sqrt{20}}\right) - P\left(Z < \frac{180 - 182}{14.7/\sqrt{20}}\right) \\ &= P(Z < 0.91) - P(Z < -0.61) \\ &= 0.8186 - 0.2709 = 0.5477. \end{aligned}$$

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(b) As $\mu = 170$, $\sigma = 26.8$, and $n = 40$, we have

$$\begin{aligned} P(180 < \overline{X} < 185) &= P(\overline{X} < 185) - P(\overline{X} < 180) \\ &= P\left(Z < \frac{185-170}{26.8/\sqrt{40}}\right) - P\left(Z < \frac{180-170}{26.8/\sqrt{40}}\right) \\ &= P(Z < 3.54) - P(Z < 2.36) \\ &= 1 - 0.9909 = 0.0091. \end{aligned}$$

□

Problem (5.6.4).

Solution. Here $\sigma = 3.4$, the desired margin of error is $E = 1$, and $\alpha = 0.1$, so

$$n = \left\lceil \left(\frac{Z_{1-\alpha/2}\sigma}{E}\right)^2 \right\rceil = \left\lceil \left(\frac{1.645 \cdot 3.4}{1}\right)^2 \right\rceil = \lceil 31.3 \rceil = 32.$$

□

Problem (5.6.5).

Solution. Here $n = 15$, $\overline{X} = 27$, $s = 4.2$, $\alpha = 0.1$. We use t with df $= 14$ rather than Z because the sample size is small. The confidence interval is given by

$$\overline{X} \pm t_{1-\alpha/2}\frac{s}{\sqrt{n}} = 27 \pm 1.761\frac{4.2}{\sqrt{15}} = 27 \pm 1.91,$$

or

$$(25.09, 28.91).$$

□

Problem (5.6.6).

Solution. Here $n = 40$ (so we use Z), $\overline{X} = 14.6$, $s = 2.8$, $\alpha = 0.05$. The confidence interval is

$$\overline{X} \pm Z_{1-\alpha/2}\frac{s}{\sqrt{n}} = 14.6 \pm 1.96\frac{2.8}{\sqrt{40}} = 14.6 \pm 0.87,$$

or

$$(13.73, 15.47).$$

□

Problem (5.6.17).

Solution. $n = 10$.

(a) $\overline{X} = 7.2$.

(b) $s = 3.9$.

(c) Use t with df $= 9$.

$$\overline{X} \pm t_{1-\alpha/2}\frac{s}{\sqrt{n}} = 7.2 \pm 2.262\frac{3.9}{\sqrt{10}} = 7.2 \pm 2.8,$$

or

$$(4.4, 10).$$

(d)

$$n = \left\lceil \left(\frac{Z_{1-\alpha/2} s}{E} \right)^2 \right\rceil = \left\lceil \left(\frac{1.96 \cdot 3.9}{1} \right)^2 \right\rceil = \lceil 58.4 \rceil = 59.$$

□

Problem (5.6.22).

Solution. It is given that $\mu = 35.2$, $\sigma = 8.8$.

(a)

$$P(\overline{X} > 38) = P\left(Z > \frac{38 - 35.2}{8.8/\sqrt{50}} \right) = P(Z > 2.25) = P(Z < -2.25) = 0.0122.$$

(b) (i) $H_0 : \mu = 35.2$, $H_1 : \mu > 35.2$, $\alpha = 0.05$.
(ii) $Z = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$.
(iii) Reject H_0 iff $Z \geq 1.645$.
(iv) $Z = \frac{38-35.2}{8.8/\sqrt{50}} = 2.25$.
(v) The data provides sufficient evidence to conclude that the mean ratio of students to professors is higher than 35.2, $\alpha = 0.05$.

□

Problem (5.6.23).

Solution. $n = 48$.

(i) $H_0 : \mu = 18$, $H_1 : \mu < 18$, $\alpha = 0.05$.
(ii) $Z = \frac{\overline{X}-\mu_0}{s/\sqrt{n}}$.
(iii) Reject H_0 iff $Z \leq -1.645$.
(iv) $Z = \frac{16.4-18}{4.1/\sqrt{48}} = -2.7$.
(v) The data provides sufficient evidence to conclude that the average daily iron intake for females aged under 51 is less than 18 mg.

□

Problem (5.6.27). **Run a two-sided test.**

Solution. $n = 15$, so we will use t with $\mathrm{df} = 14$.

(i) $H_0 : \mu = 3$, $H_1 : \mu \neq 3$, $\alpha = 0.05$.
(ii) $t = \frac{\overline{X}-\mu_0}{s/\sqrt{n}}$, $\mathrm{df} = 14$.
(iii) Reject H_0 iff $t > 2.145$ or $t < -2.145$.
(iv) $t = \frac{3.9-3}{0.4/\sqrt{15}} = 8.71$.
(v) The data provides sufficient evidence to conclude that the average time between CD4 test is significantly different from 3 months.

□