Homework 4

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Be first to report a math error for extra credit.

Read Stewart sections 2.6, 2.7. Alternatively, read
http://en.wikibooks.org/wiki/Calculus/Differentiation/Differentiation_Defined.
Both sources contain ample selections of practice exercises.

Exercise 1. Define what it means for a function $f(x)$ to be differentiable at a point a.

Exercise 2. Find the equation of the tangent line to the curve $y(x) = 4x - 3x^2$ at the point $(2, -4)$.

$[y = -8x + 12]$

Exercise 3. Find the equation of the tangent line to the curve $y(x) = \sqrt{x}$ at the point $(1, 1)$.

$[y = \frac{1}{2}x + \frac{1}{2}]$

Exercise 4. The ball is thrown into the air with a velocity of 40 m/s. Its height in meters after t seconds is given by $h(t) = 40t - 16t^2$. Find the velocity of the ball when $t = 2$.

$[-24 \text{ m/s}]$

Exercise 5. The displacement in meters of a particle moving in a straight line is given by the equation $s = 1/t^2$, where t is measured in seconds. Find the velocity of the particle at times $t = a$, $t = 1$, $t = 2$, $t = 3$.

$[-2/a^3 \text{ m/s}, -2 \text{ m/s}, -\frac{1}{4} \text{ m/s}, -\frac{2}{27} \text{ m/s}]$

Exercise 6. Find an equation of the tangent line to the graph of the function $g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

$[y = 4x - 23]$

Use the definition of the derivative to find $f'(a)$.

Exercise 7. $f(x) = 3x^2 - 4x + 1$.

$[f'(a) = 6a - 4]$

Exercise 8. $f(t) = \dfrac{2t+1}{t+3}$.

$[f'(a) = \frac{5}{(a+3)^2}]$

Exercise 9. $f(x) = \sqrt{1-2x}$.

$[f'(a) = \frac{-1}{\sqrt{1-2a}}]$

Exercise 10. Sketch the graph of a function for which all of the following conditions hold:

- $f(0) = 0$,
- $f'(0) = 3$,
- $f'(1) = 0$,
- $f'(2) = -1$.

Find the derivative of the given function by using the definition of the derivative. Find the respective domains of the function and of its derivative.

Exercise 11. $f(x) = \frac{1}{2}x - \frac{1}{3}$.

$[f'(x) = \frac{1}{2}, \mathbb{R}, \mathbb{R}]$

Exercise 12. $f(x) = \sqrt{1+2x}$.

$[f'(x) = \frac{1}{\sqrt{1+2x}}, x \geqslant -\frac{1}{2}, x > -\frac{1}{2}]$

Exercise 13. $g(t) = \dfrac{4t}{t+1}$.

$[g'(t) = \frac{4}{(t+1)^2}, t \neq -1, t \neq -1]$