FINAL EXAM

2012-06-27

NAME: ____

This test is closed books, closed notes. Read through the entire thing first and distribute your time wisely. Fully justify your answers and show all work in order to maximize your partial credit.

This test has 160 points in ten (10) problems. Make sure you have all the pages right away.

1 (20 points)**.** True or False? (No need to show work.)

(1) For any function of two variables $f(x, y)$, $f_{xy} = f_{yx}$.
False

(2) For any field $\mathbf{F}$ in $\mathbb{R}^3$, $\nabla \cdot (\nabla \times \mathbf{F})$ is a vector field.
False

(3) For any field $\mathbf{F}$ in $\mathbb{R}^3$, $\nabla \times (\nabla \cdot \mathbf{F})$ is a scalar field.
False

(4) In a conservative vector field, a line integral depends on the path taken.
False

(5) If a surface in $\mathbb{R}^3$ can be parametrized by a function of two variables $\mathbf{r}(u, v)$, then the parametrization is unique.
False

2 (10 points)**.** Describe the domain of the function

$$f(x,y) = \frac{\sqrt{1-x^2-y^2}}{\ln(x+y)}.$$

The set of points satisfying $x^2+y^2 \leq 1$, $y > -x$, and $y \neq 1-x$.

3 (10 points)**.** Find the gradient field for the function $\phi(x, y, z) = \dfrac{1}{|\mathbf{r}|}$, where $\mathbf{r}(x, y, z) = \langle x, y, z\rangle$.

$$\frac{-\mathbf{r}}{|\mathbf{r}|^3} = \frac{-\langle x, y, z\rangle}{(x^2 + y^2 + z^2)^{3/2}}$$

4 (20 points)**.** Evaluate the line integral

$$\int_C ye^{-xz}\,ds,$$

where C is the segment

$$\mathbf{r}(t) = \langle t, 3t, -6t\rangle,\ t \in [0, \ln 8].$$

$$\frac{\sqrt{46}}{4}\left(e^{6(\ln 8)^2} - 1\right)$$

5 (20 points)**.** Let the force field in $\mathbb{R}^3$ be given by

$$\mathbf{F}(x, y, z) = \langle -y, z, x \rangle.$$

Find the work required to move an object in this force field along the path consisting of a line segment from $(0, 0, 0)$ to $(0, 1, 0)$, followed by a line segment from $(0, 1, 0)$ to $(0, 1, 4)$.

0

6 (10 points)**.** Determine whether the field

$$\mathbf{F} = \langle y, x + z^2, 2yz \rangle$$

is conservative. If yes, find its potential function.

$\phi(x, y, z) = xy + yz^2$

7 (10 points). Let $\mathbf{F} = \langle y, x + z^2, 2yz \rangle$ (the same as in the previous problem). Find the work done by this force field as an object moves along the path

$$\mathbf{r}(t) = \langle \ln(1+t), e^t, t^2 + 1 \rangle, \;\; t \in [0,1].$$

$e \ln 2 + 4e - 1$

8 (20 points)**.** Compute the divergence and the curl of the field

$$\mathbf{F}(x,y,z) = \langle 2xy + z^4, x^2, 4xz^3 \rangle.$$

$\nabla \cdot \mathbf{F} = 2y + 12xz^2$, $\nabla \times \mathbf{F} = \mathbf{0}$

9 (20 points)**.** Find the flux of the field $\mathbf{F} = \langle x, y, z \rangle$ across the curved side of the cylinder $x^2 + y^2 = 1$, $|z| \leq 8$, with the normal vector pointing outwards.

32π

10 (20 points)**.** Evaluate both integrals in the statement of the Green's Theorem (circulation form) for the field $\mathbf{F} = \langle 2y, -2x \rangle$ and the region bounded by $y = \sin(x)$ and $y = 0$, for $x \in [0, \pi]$.

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