TEST 1

2012-06-06

NAME: _____

This test is closed books, closed notes. Read through the entire thing first and distribute your time wisely. Fully justify your answers and show all work in order to maximize your partial credit.

1 (20 points)**.** True or False?

(1) For all real numbers x, y, z,

$$\langle x, y\rangle \cdot \langle -y, x\rangle = |\langle x, y, z\rangle \times \langle x, y, z\rangle|.$$

True

(2) $\langle x, y\rangle \times \langle x, y\rangle = 0$ for all real numbers x and y.

False

(3) $\mathbf{v} \cdot \mathbf{v} = |\mathbf{v}|^2$ for all vectors $\mathbf{v}$.

True

(4) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$ for all non-zero vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$.

True

(5) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ for all non-zero vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$.

False

2 (10 points)**.** Find the area of a parallelogram with sides corresponding to vectors $\langle 1, -2, 3\rangle$ and $\langle -2, 1, -1\rangle$.

$\sqrt{35}$

3 (10 points)**.** Find a vector equation or parametric equations of the line where the planes $2x + y - z = 1$ and $-x + 3y - z = 4$ intersect.

$\mathbf{r}(t) = \langle 1, 3, 4 \rangle + t\langle 2, 3, 7 \rangle$

4 (10 points). Find the angle between the planes $2x+y-z=1$ and $-x+3y-z=4$. (Leaving it as $\cos^{-1}$ is OK.)

$\cos^{-1}\left(\frac{2}{\sqrt{66}}\right)$

5 (10 points). Find a vector $\mathbf{v}=\langle a,b,c\rangle$ such that

$$\mathbf{v}\times\langle 1,2,3\rangle=\langle 2,-4,2\rangle,$$

or show it doesn't exist.

$\langle 0,-2,-4\rangle$

6 (10 points). Find a vector $\mathbf{v} = \langle a, b, c \rangle$ such that

$$\mathbf{v} \times \langle 1, 2, 3 \rangle = \langle 1, 0, 0 \rangle,$$

or show it doesn't exist.

Does not exist

7 (10 points). Find the length of the curve

$$\mathbf{r}(t) = \left\langle \frac{2}{3}(t-1)^{3/2}, \sin(t), \cos(t) \right\rangle$$

as t ranges from 1 to 4.

$\frac{14}{3}$