# TEST 2

2012-06-20

NAME: _____

This test is closed books, closed notes. Read through the entire thing first and distribute your time wisely. Fully justify your answers and show all work in order to maximize your partial credit.

This test has 100 points[1] in six (6) problems. Make sure you have all the pages right away.

[1] There is a singularity hiding in this test. Identify it for some extra credit.

**1** (10 points). Find the limit

$$\lim_{(x,y)\to(-1,1)} \frac{x^2 - y^2}{x^2 - xy - 2y^2}$$

or prove it does not exist.

$\frac{2}{3}$

**2** (20 points)**.** Identify all critical points of the function

$$f(x, y) = x^4 + y^4 - 16xy.$$

Use the second derivative test to determine whether each critical point corresponds to a local maximum, a local minimum, or a saddle.

Local minima and $(2, 2)$ and $(-2, -2)$, saddle at $(0, 0)$

**3** (10 points). Find the equation of the plane tangent to the surface

$$x = \ln(1 + zy)$$

at the point $(\ln(3), 2, 1)$.

$x - \ln(3) = \frac{1}{3}(y-2) + \frac{2}{3}(z-1)$

**4** (20 points). Find the value of

$$\int_0^1 \int_{\sqrt[3]{y}}^1 x^{10} \cos(\pi x^4 y) dx dy.$$

$\frac{2}{7\pi^2}$

**5** (20 points)**.** Find the mass of the apple-shaped object

$$\{(\rho, \phi, \theta) \mid \rho \leq 3 + 2\cos(\phi)\}$$

if the density is given by

$$\delta(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$$

$12\pi$

**6** (20 points). Find the volume of the shape bounded by the planes $z = 0$, $x = 0$, $x = y$, $y = \pi$, and the surface $z = \sin(x)$.

$\pi$