

TEST 1

2013-07-09

NAME: _____

This test is closed books, closed notes. Read through the entire thing first and distribute your time wisely. Fully justify your answers and show all work in order to maximize your partial credit.

This test has 70 points in seven (7) problems. Make sure you have all the pages right away.

1 (10 points). Find all unit vectors perpendicular to \mathbf{v} and all unit vectors parallel to \mathbf{v} if

$$\mathbf{v} = \langle -1, 2 \rangle$$

Parallel: $\pm\langle -1, 2 \rangle / \sqrt{5}$; perpendicular: $\pm\langle 2, 1 \rangle / \sqrt{5}$

2 (10 points). Find the volume of the parallelepiped with edges given by vectors $\langle 1, 1, 1 \rangle$, $\langle 1, 1, -1 \rangle$, and $\langle 1, -1, -1 \rangle$.

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3 (10 points). Find the angle between the curves

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle \text{ and}$$

$$\mathbf{u}(s) = \langle e^{s-2}, \ln(s-1), \pi s \rangle$$

at the point of intersection $(1, 0, 2\pi)$. It is OK to leave the answer as \cos^{-1} .

$$\cos^{-1} \left(\frac{1 + \pi}{\sqrt{4 + 2\pi^2}} \right)$$

4 (10 points). Find the length of the curve $\mathbf{r}(t) = \langle t, 8 \sin(t), 8 \cos(t) \rangle$ for $t \in [0, 4\pi]$.

$$4\sqrt{65}\pi$$

5 (10 points). Find an equation of the plane containing the point $(1, 2, 3)$ and perpendicular to both of the following planes:

$$x + y + z = 1$$

$$x - 2y - 3z = 1$$

$$(x - 1) - 4(y - 2) + 3(z - 3) = 0$$

6 (10 points). Show that the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x+2y}{x-2y}$ does not exist.

The limit is 1 along the line $y = 0$ and -1 along the line $x = 0$.

7 (10 points). Find all four second order partial derivatives of the function $f(x, y) = (y^2 + 1)e^x$.

$$f_{xx} = f, f_{yy} = 2e^x, f_{xy} = f_{yx} = 2ye^x$$