# LHW1 Key

## Vectors

(4) 4pts

$\bar{a} = 2\bar{i} - 4\bar{j} + 4\bar{k} = \langle 2, -4, 4 \rangle$

$\bar{b} = 2\bar{j} - \bar{k} = \langle 0, 2, -1 \rangle$

JUST Answers OK

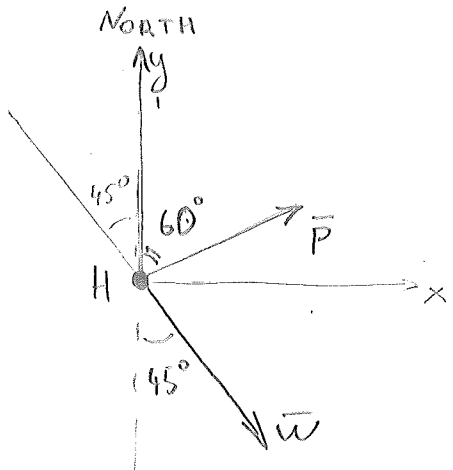
$\bar{a} + \bar{b} = \boxed{\langle 2, -2, 3 \rangle}$

$2\bar{a} + 3\bar{b} = \langle 4, -8, 8 \rangle + \langle 0, 6, -3 \rangle = \boxed{\langle 4, -2, 5 \rangle}$

$|\bar{a}| = \sqrt{2^2 + 4^2 + 4^2} = \sqrt{4+16+16} = \boxed{6}$

$|\bar{a} - \bar{b}| = |\langle 2, -6, 5 \rangle| = \sqrt{4 + 36 + 25} = \boxed{\sqrt{65}}$

(7) 2pts




$|\bar{p}| = 80$ , $|\bar{w}| = 50$

$\bar{p} = \langle 80 \cdot \cos\frac{\pi}{6}, 80 \cdot \sin\frac{\pi}{6} \rangle$

$\bar{w} = \langle 50\cos\left(-\frac{\pi}{4}\right), 50 \cdot \sin\left(-\frac{\pi}{4}\right) \rangle$

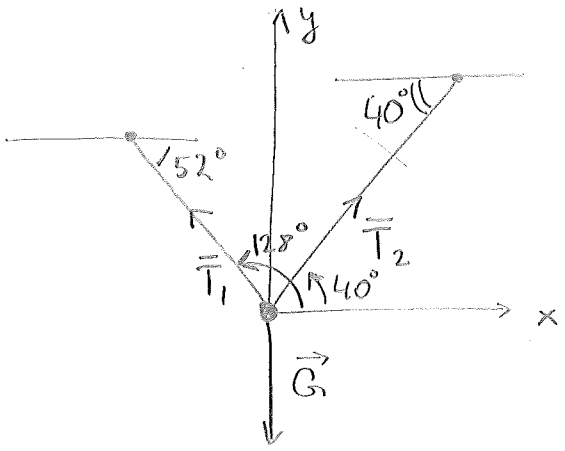
$\bar{p} + \bar{w} = \langle 80\frac{\sqrt{3}}{2} + 50\frac{\sqrt{2}}{2}, 40 - 50\frac{\sqrt{2}}{2} \rangle$

$\text{true course} = \text{atan}\left(\frac{40 - 50\frac{\sqrt{2}}{2}}{80\frac{\sqrt{3}}{2} + 50\frac{\sqrt{2}}{2}}\right) \approx \boxed{2.54^\circ} \rightleftharpoons \boxed{N87^\circ E}$

$\text{ground speed} = |\bar{p} + \bar{w}| \approx \boxed{104.74} \frac{km}{h}$

8

2 pts




Just check magnitudes

From physics: $\vec{T_1} + \vec{T_2} + \vec{G} = \vec{0}$ ☆

$\vec{G} = \langle 0, mg \rangle = \langle 0, -5 \cdot 9.8 \rangle$

$m_1, m_2$ magnitudes of $\vec{T_1}, \vec{T_2}$

☆ $\begin{cases} m_1 \sin 128^\circ + m_2 \sin 40^\circ = +5 \cdot 9.8 & (Y \text{ comp}) \\ m_1 \cos 52^\circ = m_2 \cos 40^\circ & (X \text{ comp}) \end{cases}$

$$m_2 = \frac{m_1 \cos 52^\circ}{\cos 40^\circ}$$

$$m_1 \sin 128^\circ + \frac{m_1 \cos 52^\circ}{\cos 40^\circ} \cdot \sin 40^\circ = +5 \cdot 9.8$$

$$m_1 = \frac{+5 \cdot 9.8}{\sin 128^\circ + \cos 52^\circ \cdot \tan 40^\circ} \approx \boxed{37.56}$$

$$m_2 \approx \boxed{30.19}$$

Three-Dim. Coord. Sys.

4 (2 pts)

$4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$

$4(x^2 - 2x + 1) + 4(y^2 + 4y + 4) + 4z^2 = 1 + 4 + 16$

Show SOME work

$4(x-1)^2 + 4(y+2)^2 + 4z^2 = 21$

$(x-1)^2 + (y+2)^2 + z^2 = \frac{21}{4}$

sphere with radius $\frac{\sqrt{21}}{2}$ centered at $(1, -2, 0)$

5 (2 pts)

SOME work

center is in the middle: $[\langle 2,1,4\rangle + \langle 4,3,10\rangle]/2 = \langle 3,2,7\rangle$

radius is $|\langle 3,2,7\rangle - \langle 2,1,4\rangle|$

$= |\langle 1,1,3\rangle| = \sqrt{1+1+9} = \sqrt{11}$

$(x-3)^2 + (y-2)^2 + (z-7)^2 = 11$

7
2 pts

$$x^2 + y^2 + z^2 > 2z$$

$$x^2 + y^2 + z^2 - 2z + 1 > 1$$

$$x^2 + y^2 + (z-1)^2 > 1$$

outside of the sphere of rad. 1 centered at $\langle 0, 0, 1 \rangle$.

some WORK

8
2 pts

Y points UP

$$\text{and} \begin{cases} y \geq 0 \\ x^2 + y^2 + z^2 \leq 4 \end{cases}$$

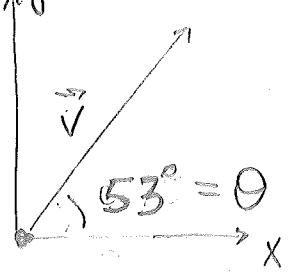
## DOT Product

2
1 pt

$$6 \cdot \frac{1}{3} \cdot \cos\frac{\pi}{4} = \frac{6}{3} \cdot \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$$

Answer OK

8
2pts




$$\theta = \operatorname{atan}\left(\frac{4}{3}\right) \approx 53°.$$

$$\theta_1 = 53° + 60° = \boxed{113°}$$

$$\theta_2 = 53° - 60° = \boxed{-7°}$$

} Ok

$$\bar{x}_1 = \langle \cos 113°, \sin 113° \rangle \approx \langle -0.39, 0.92 \rangle$$

$$\bar{x}_2 = \langle \cos -7°, \sin -7° \rangle \approx \langle 0.99, -0.12 \rangle$$

# Cross Product

**2** (2 pts)

$$|\bar{u}\times\bar{v}| = |\bar{u}||\bar{v}|\sin\theta = 5\cdot 10\cdot\frac{\sqrt{3}}{2} = \boxed{25\sqrt{3}}$$

Answer Ok

into the page

**6** (2 pts)

$$\langle 1,1,1\rangle \times \langle 2,0,1\rangle =$$

$$= -\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \langle 1, 1, -2\rangle$$

unit vectors: $\dfrac{\pm\langle 1,1,-2\rangle}{|\langle 1,1,-2\rangle|} = \boxed{\dfrac{\pm\langle 1,1,-2\rangle}{\sqrt{6}}}$

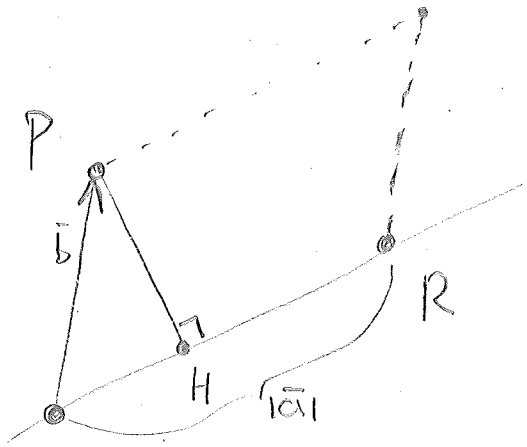
some work

**9** (1 pt)

$|\overline{PH}|$ is the hight of a parallelogram with area $|\bar{a}\times\bar{b}|$, so

$$|\overline{PH}| = \frac{|\bar{a}\times\bar{b}|}{|\bar{a}|}$$

height ↑ base ↑ area




$\bar{a} = \overline{QR}$

$b = \overline{QP}$

SOME work.