Arc length & Curvature

LHW3-KEY

5

3 pts

Parametrize $\bar{r}(t) = \langle e^{2t}\cos(2t), 2, e^{2t}\sin(2t) \rangle$

$$\int_0^t |r'(u)|\,du = \int_0^t |\langle 2e^{2u}\cos(2u) - 2e^{2u}\sin(2u), 0,$$

$$2e^{2u}\sin(2u) + 2e^{2u}\cos(2u) \rangle|\,du =$$

$$= \int_0^t \sqrt{8e^{4u}\cos^2(2u) + 8e^{4u}\sin^2(2u)}\,du$$

show some work

$$= \int_0^t \sqrt{8e^{4u}}\,du = \int_0^t 2\sqrt{2}\,e^{2u}\,du$$

$$= \sqrt{2}\int_0^t e^{2u}\,d2u = \sqrt{2}\,e^{2u}\Big|_0^t = \sqrt{2}(e^{2t} - 1) = s$$

so $e^{2t} - 1 = \frac{s}{\sqrt{2}}$

$$e^{2t} = 1 + \frac{s}{\sqrt{2}}$$

$$2t = \ln\left(1 + \frac{s}{\sqrt{2}}\right)$$

$$t = \frac{1}{2}\ln\left(1 + \frac{s}{\sqrt{2}}\right)$$

$$\bar{r}(s) = \left\langle \left(1 + \frac{s}{\sqrt{2}}\right)\cos\left[\ln\left(1 + \frac{s}{\sqrt{2}}\right)\right], 2, \left(1 + \frac{s}{\sqrt{2}}\right)\sin\left[\ln\left[1 + \frac{s}{\sqrt{2}}\right]\right] \right\rangle$$

Equations of lines & planes

2 2 pts

2 points on the line $\vec{r}(t) = \langle 2t, 1-t, 4+3t \rangle$

$t = 0$: $(0, 1, 4)$

$t = 1$: $(2, 0, 7)$

show some work

a vector $\parallel$ to the line is

$\langle 2, -1, 3 \rangle$, so

$\vec{r}(t) = \langle 0,0,0 \rangle + t\langle 2,-1,3 \rangle = \boxed{t\langle 2, -1, 3 \rangle}$

14 2 pts

normal vector :

$\vec{n} = \langle 2, -4, 6 \rangle \times \langle 5, 1, 3 \rangle =$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -4 & 6 \\ 5 & 1 & 3 \end{vmatrix} = \langle -18, 24, 22 \rangle = 2\langle -9, 12, 11 \rangle$$

so equation is

$$\boxed{-9x + 12y + 11z = 0}$$

some work

15

2 pts

direction of the line:

$\langle 4, -2, 2\rangle - \langle 1, 0, 1\rangle = \langle 3, -2, 1\rangle$.

equations of the line:

$$\begin{cases} x = 1 + 3t \\ y = 0 - 2t \\ z = 1 + t \end{cases}$$

combine this with

$x + y + z = 6$:

some work

$$1 + 3t - 2t + 1 + t = 6$$

$$\Rightarrow 2t = 4 \quad \Rightarrow t = 2$$

intersection at $\bar{r}(2) = \boxed{\langle 7, -4, 3\rangle}$

17

2pts

$\langle 1, 2, 2\rangle \neq c \cdot \langle 2, -1, 2\rangle$, so not parallel.

Angle θ can be found from dot product:

some work

$$\langle 1, 2, 2\rangle \circ \langle 2, -1, 2\rangle = |\langle 1, 2, 2\rangle| |\langle 2, -1, 2\rangle| \cos\theta$$

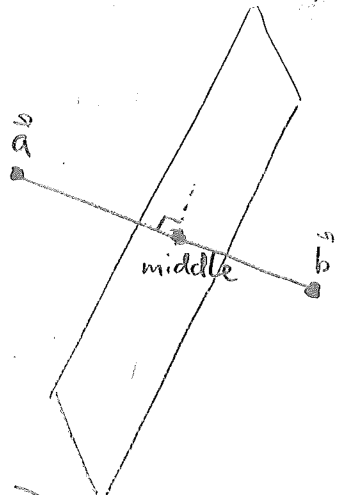
$$2 - 2 + 4 = 3 \cdot 3 \cdot \cos\theta$$

$$\cos\theta = \frac{4}{9}$$

$$\theta = a\cos\frac{4}{9} \approx 1.11 \approx 63.61^\circ$$

18
2 pts

$\vec{a} = \langle -4, 2, 1 \rangle$, $\vec{b} = \langle 2, -4, 3 \rangle$



normal vector:

$\vec{n} = (\vec{b} - \vec{a})/2 = \langle 3, -3, 1 \rangle$

midpoint:

$\vec{m} = (\vec{b} + \vec{a})/2 = \langle -1, -1, 2 \rangle$

so the equation of the plane is

$$\boxed{\begin{array}{l} 3(x+1) - 3(y+1) + (z-2) = 0 \\ \text{or} \quad 3x - 3y + z = 2 \end{array}}$$

Partial Derivs

4
2 pts

$f_y(x,y) = 3\cos(2x + 3y)$

$f_y(-6, 4) = 3\cos(-12 + 12) = \boxed{3}$

5

2pts

Checking that $u_{xy} = u_{yx}$.

$$u_x = 4x^3y^2 - 2y^5$$

$$u_{xy} = 8x^3y - 10y^4$$

some work

$$u_y = 2x^4y - 10xy^4$$

$$u_{yx} = 8x^3y - 10y^4 = u_{xy} \quad \square$$

Chain Rule

3

2 pts.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial s} + \frac{\partial z}{\partial \beta}\cdot\frac{\partial \beta}{\partial s} =$$

$$= \cos\alpha \cdot \tan\beta \cdot 3 + \frac{1}{\cos^2\beta}\cdot \sin\alpha \cdot 1$$

$$\boxed{= 3\cos\alpha \cdot \tan\beta + \frac{\sin\alpha}{\cos^2\beta}} = 3\cos(3s+t)\tan(s-t) + \frac{\sin(3s+t)}{\cos^2(s-t)}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial t} + \frac{\partial z}{\partial \beta}\cdot\frac{\partial \beta}{\partial t} =$$

$$= \boxed{\cos\alpha \cdot \tan\beta - \frac{\sin\alpha}{\cos^2\beta}} = \cos(3s+t)\tan(s-t) - \frac{\sin(3s+t)}{\cos^2(s-t)}$$

answers in terms of α, β OK

10

1 pts

$$\frac{dW}{dt} = \frac{\partial W}{\partial T} \cdot \frac{dT}{dt} + \frac{\partial W}{\partial R} \cdot \frac{dR}{dt}$$

$$= -2 \cdot 0.15 + 8 \cdot (-0.1)$$

$$= \boxed{-1.1}$$

If we convert to meters, though, it's $\boxed{-0.308}$