MA 225 TEST 2 KEY

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Problem 1. *(10 points)* Find the set of critical points of the function

$$f(x,y) = x^2 e^{-y^2}.$$

[Hint: the set of critical points may be infinite.]

$f_x = 2xe^{-y^2}$, $f_y = -2x^2ye^{-2y^2}$, and any point $(0,y)$ is a solution. In set builder notation, the set of critical points is

$$\{(0,y) : y \in \mathbb{R}\}.$$

Problem 2. Let $f(x,y) = x^2y - y^2 + xy$.

(a) *(15 points)* Find the set of critical points of f.

$f_x = 2xy + y$, $f_y = x^2 - 2y + x$. If $f_x = 0$ then either $y = 0$ or $x = -1/2$. If $y = 0$ and $f_y = 0$ then $x = 0$ or $x = -1$. If $x = -1/2$ and $f_y = 0$ then $y = -1/8$. So there are 3 critical points: $(0,0)$, $(-1,0)$, and $(-1/2,-1/8)$.

(b) *(15 points)* For each critical point, run the second derivative test to determine whether the point is a local maximum, a local minimum, a saddle node, or a mystery point.

$f_{xx} = 2y$, $f_{yy} = -2$, $f_{xy} = 2x + 1$.
$D(0,0) = -1$, so it's a saddle node.
$D(-1,0) = -1$, so it's a saddle node.
$D(-1/2,-1/8) = 1/2$ while $f_{xx}(-1/2,-1/8) = -1/4$, so it's a local maximum.

Problem 3. *(10 points)* Find absolute maxima and minima of the function

$$f(x,y) = xy$$

on the plane region

$$R = \{(x,y) : 0 \le x \le 2y - y^2 \text{ and } 0 \le y \le 2\}.$$

[Hint: either set may be infinite.]

$f_x = y$, $f_y = x$, so f has a single critical point $(0,0)$.
The boundary on the left is $x = 0$, $y \in [0,2]$. There $f(0,y) = 0$.

Date: 2012-06-21.

The boundary on the right is $x = y(2-y)$. There

$$h(y) = f(y(2-y), y) = y^2(2-y),$$
$$h'(y) = y(4-3y),$$

so h has a critical point at $y = 4/3$.

Hence all the points on the left boundary

$$\{(0,y) : y \in [0,2]\}$$

are global minima with $f(0,y) = 0$, and the point $(8/9, 4/3)$ is a global maximum with $f(8/9, 4/3) = 32/27$.

Problem 4. Let $f(x,y) = x + 2y$ and let

$$I = \iint_R f(x,y)dA,$$

where R is the region below the x-axis and above the parabola $y(x) = x^2 - 1$.

(a) *(10 points)* Write down the iterated integral for I in two different ways corresponding to two different orders of integration.

$$I = \int_{-1}^{1}\int_{x^2-1}^{0} f(x,y)dydx = \int_{-1}^{0}\int_{-\sqrt{y+1}}^{\sqrt{y+1}} f(x,y)dxdy.$$

(b) *(10 points)* Evaluate the integral I.

$I = -16/15$.

Problem 5. Let

$$I = \iint_R \frac{1}{x^2+y^2}dA,$$

where R is the closed plane region in the second quadrant between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(a) *(10 points)* Write the integral I as an iterated integral in polar coordinates.

$$I = \int_{\pi/2}^{\pi}\int_{1}^{2} \frac{1}{r}drd\theta.$$

(b) *(10 points)* Evaluate the integral I.

$$I = \frac{\pi \ln 2}{2}.$$