

CARDINALITY HOMEWORK

BOOK OF PROOF: 14.1 - 14.3

LAST NAME	FIRST NAME	DATE
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In the following exercises, determine the cardinality of each set, and which one is larger, or whether their cardinalities are the same.

1. W = the set of days in a week, and B = the set of months in a year.
2. B = the set of letters used to spell the word “banana”, and T = the set of letters used to spell the word “totoro”.
3. $X = \{1, 2, 3\}$, and $Y = \mathcal{P}(X)$.
4. E = all non-negative even integers, and D = all positive odd integers.
5. N = all negative integers, and $T = \{10, 20, 30, 40, \dots\}$ all positive integer multiples of ten.
6. P = prime numbers between 2 and 100, and $\mathbb{Q}$ = all rational numbers.
7. $\mathbb{Z}$ = all integers, and I = all rational numbers between 0 and 1.
8. J = all irrational numbers, and $\mathbb{R}$ = all real numbers.
9. I = all real numbers between -10 and 10 , and N = all negative rational numbers.
10. $\mathbb{R}$ = all real numbers, and $P = \mathcal{P}(\mathbb{R})$.
11. S = the set of positive integer divisors of the number 60, and E = the set of positive integer divisors of the number 80.
12. T = the set of all prime numbers between 1 and 20, and R = the set of all integers between 1 and 20 which are divisible by 3.

In the following exercises, show that two sets have the same cardinality by thinking up a rule for pairing up their elements. State the rule by making paired lists, and/or by writing a formula relating the elements of two sets.

13. T = the set of positive integer multiples of 20,
and F = the set of positive integer multiples of 50.

14. S = positive integers that end with digit 6,
and E = positive integers that end with digit 8.

15. N = negative integers,
and S = squares of integers.

16. $G = \{x \mid x \text{ is an integer and } x \leq -7\}$,
and $H = \{x \mid x \text{ is an integer and } x \geq 9\}$.

17. D = the set of all positive integers which use no digits other than 4,
and J = the set of all positive integers which use no digits other than 3.

18 (†). E = the set of all even integers,
and $\mathbb{N}$ = the set of all non-negative integers.

ANSWERS

1. B is larger with $12 > 7$.
3. Y is larger with $8 > 3$.
5. The same cardinality $\aleph_0$.
7. The same cardinality $\aleph_0$.
9. I is larger with $\mathfrak{c} > \aleph_0$.
11. S is larger with $12 > 10$.
13. $T = \{20, 40, 60, 80, \dots\}$, $F = \{50, 100, 150, 200, \dots\}$, $f = 5t/2$
15. $N = \{-1, -2, -3, -4, -5, -6, \dots\}$, $S = \{0, 1, 4, 9, 16, 25, \dots\}$, $s = (n + 1)^2$
17. $D = \{4, 44, 444, 4444, \dots\}$, $J = \{3, 33, 333, 3333, \dots\}$, $j = 3d/4$