

EQUIVALENT STATEMENTS HOMEWORK

TEXT: 2.4, 2.5, 2.6

LAST NAME	FIRST NAME	DATE
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For each pair of statements, determine whether they are equivalent, and how they relate to each other: are they mutual converses, inverses, or contrapositives?

1.

- If I am not a US citizen, then I cannot vote in federal elections.
- If I am a US citizen, then I can vote in federal elections.

2.

- If I am drunk, then I am not allowed to drive.
- If I am allowed to drive, then I am not drunk.

3.

- If you have nothing left to lose, then you don't have freedom.
- If you have freedom, then you have nothing left to lose.

4.

- If the water is wet, then water is not wet.
- If the water is not wet, then water is wet.

5. Rewrite the given statement as a disjunction, and its negation as a conjunction:

If something is not broken, then we should not fix it.

As a disjunction:

Negation as a conjunction:

For each of the following statements, construct side-by-side truth tables, fill out answer columns completely, and box them. Also, for each pair of statements, determine whether they are equivalent.

6.

$$(x \wedge y) \vee ((\sim x) \wedge (\sim y))$$

$$x \leftrightarrow y$$

Equivalent

Not equivalent

7.

$$(A \leftrightarrow B) \vee C$$

$$(A \leftrightarrow C) \vee (B \leftrightarrow C)$$

Equivalent

Not equivalent

8. Define the connective $A \oplus B$, known as *exclusive or* or *XOR*, by the following truth table:

A	B	$A \oplus B$
T	T	F
T	F	T
F	T	T
F	F	F

Find a statement using any of the five connectives defined so far, which is equivalent to $A \oplus B$.

9. Define the connective $A \# B$, known as *NAND*, by the following truth table:

A	B	$A \# B$
T	T	F
T	F	T
F	T	T
F	F	T

Find a statement using any of the five connectives defined so far, which is equivalent to $A \# B$.

10. Construct a truth table for the following statement:

$$(x \oplus (y \# x)) \# (y \oplus x)$$

Use truth tables to determine whether the two statements are equivalent.

11. $p \vee (q \wedge (\sim p))$ versus $q \vee p$

Equivalent

Not equivalent

12. P versus $\sim P \rightarrow (Q \wedge (\sim Q))$

Equivalent

Not equivalent

13. $\sim (a \vee b)$ versus $(\sim a) \vee (\sim b)$

Equivalent

Not equivalent

14. $\sim (a \wedge (\sim b))$ versus $(\sim a) \vee b$

Equivalent

Not equivalent

15. $((x \rightarrow y) \wedge (y \rightarrow z)) \wedge (z \rightarrow x)$ versus $(y \leftrightarrow z) \wedge (y \leftrightarrow x)$

Equivalent

Not equivalent

16. $x \rightarrow (y \vee z)$ versus $(x \rightarrow y) \vee (x \rightarrow z)$

Equivalent

Not equivalent

17. Write the converse, the inverse, and the contrapositive for the statement:

If you are under age 17, then you cannot attend this movie.

Converse:

Inverse:

Contrapositive:

18. Write the converse, the inverse, and the contrapositive for the statement:

Shaneese will go to work only if she doesn't have a fever.

Converse:

Inverse:

Contrapositive:

For each of the following conditionals,
(a) rewrite it as a disjunction and
(b) write down its negation as a conjunction.

19. *If Luke faces Vader, then Obi-Wan cannot interfere*
As a disjunction:

Negation as a conjunction:

20. *If you don't look both ways before crossing the street, then you might get hit by a car.*
As a disjunction:

Negation as a conjunction:

21. *If you don't eat your meat, you can't have any pudding!*

-Pink Floyd

As a disjunction:

Negation as a conjunction:

22. *Math is fun only if it's completely and totally useless.*

As a disjunction:

Negation as a conjunction:

23. *If you think you understand quantum mechanics, then you don't understand quantum mechanics.*

-Attributed to Richard Feynman

As a disjunction:

Negation as a conjunction:

24. Assume that the statement

If you swear, then you will get your mouth washed out with soap

is true. Which of the following statements must also be true?

- (a) If you don't swear, then you won't get your mouth washed out with soap.
- (b) If you don't get your mouth washed out with soap, then you didn't swear.
- (c) If you get your mouth washed out with soap, then you swore.

25 (†). Use $\#$ and only $\#$ to express each of the five connectives we've defined so far, as well as the *exclusive or* $\oplus$. For example, (and you should check this via a truth table) the negation $\sim A$ can be expressed as $A\#A$.

ANSWERS

1. Not equivalent, inverses.

3. Not equivalent, neither.

5.

As a disjunction: *Either something is broken or we should not fix it.*

Negation as a conjunction: *Something is not broken and we should fix it.*

7. Not equivalent:

A	B	C	$(A \leftrightarrow B)$	$\vee$	C	$(A \leftrightarrow C)$	$\vee$	$(B \leftrightarrow C)$
T	T	T		T		T		
T	T	F		T		F		
T	F	T		T		T		
T	F	F		F		T		
F	T	T		T		T		
F	T	F		F		T		
F	F	T		T		F		
F	F	F		T		T		

9. $(\sim A) \vee (\sim B)$

11. Equivalent:

p	q	$p \vee (q \wedge (\sim p))$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

13. Not equivalent:

a	b	$\sim (a \vee b)$	$(\sim a) \vee (\sim b)$
T	T	F	F
T	F	F	T
F	T	F	T
F	F	T	T

15. Equivalent:

x	y	z	$((x \rightarrow y) \wedge (y \rightarrow z)) \wedge (z \rightarrow x)$	$(y \leftrightarrow z) \wedge (y \leftrightarrow x)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	T	T

17.

Converse: *If you cannot attend this movie, then you are under age 17.*

Inverse: *If you are not under age 17, then you can attend this movie.*

Contrapositive: *If you can attend this movie, then you are not under age 17.*

19.

Disjunctive form: *Either Luke does not face Vader, or Obi-Wan cannot interfere.*

Negation as a conjunction: *Luke faces Vader, and Obi-Wan can interfere.*

21.

Disjunctive form: *Either you eat your meat, or you can't have any pudding!*

Negation as a conjunction: *You don't eat your meat, but you can have some pudding!*

23.

Disjunctive form: *Either you don't think you understand quantum mechanics, or you don't understand quantum mechanics.*

Negation as a conjunction: *You think you understand quantum mechanics, and you do understand quantum mechanics.*