SETS

TEXT: 1.1, 1.2

LAST NAME	FIRST NAME	DATE

1 (8 points)**.** Let the universe U and the sets within U be defined as follows:

$$\begin{aligned} U &= \{0,1,2,3,4,5,6,7,8,9\} \\ A &= \{2,3,5,7\} \\ B &= \{0,1,5,9\} \\ C &= \{1,9\} \end{aligned}$$

Determine whether the following statements are true or false:

(a) $A \in U$

(b) $5 \in A$

(c) $C \subseteq B$

(d) $C \subset B$

(e) $B \subset B$

(f) $B \subseteq B$

(g) $\varnothing \subseteq B$

(h) $\varnothing \in B$

2 (2 points)**.** If $D = \{\varnothing, 5\}$, state the set $\mathscr{P}(D)$ in roster notation:

3 (2 points)**.** State in roster notation the set of all proper subsets of the set $\{M, E, W\}$

4 (5 points)**.** Use the roster notation (or $\varnothing$ for the empty set) to represent the following sets.

(a) $\{x \mid x \text{ is an integer and } 99 \leq x \leq 103\}$

(b) $\{x \mid x \text{ is a negative integer and } x > -5\}$

(c) $\{x \mid x \text{ is prime and } x \leq 10\}$

(d) $\{x^2 + 1 \mid x \text{ is an integer and } -3 \leq x \leq 3\}$

(e) $\{x \div 2 \mid x \text{ is an even integer with } -6 \leq x \leq 6\}$

5. Write down the following sets using the set-builder notation.

(a) The set of positive multiples of five: $\{5, 10, 15, 20, \ldots\}$

$$\Big\{ \qquad\qquad \Big| \; x \text{ is a positive integer} \Big\}$$

(b) The set of third powers of integers: $\{0, 1, -1, 8, -8, 27, -27, 64, -64, \ldots\}$

$$\Big\{ \qquad\qquad \Big| \; x \text{ is an integer} \Big\}$$

(c) The set of integers less than or equal to -7: $\{-7, -8, -9, -10, \ldots\}$

$$\Big\{ \qquad\qquad \Big| \; x \text{ is a positive integer} \Big\}$$

(d) The set of positive integers that end with seven: $\{7, 17, 27, 37, 47, \ldots\}$

$$\Big\{ \qquad\qquad \Big| \; x \text{ is a positive integer} \Big\}$$

6. Let the universe $U = \mathbb{R}$ be the set of all real numbers, and let

$$\begin{aligned} M &= \{x \mid -10 < x < 10\} \\ L &= \{x \mid x \leq 10\} \\ R &= \{x \mid x > 1\} \\ P &= \{x \mid x \text{ is prime}\} \end{aligned}$$

Determine whether each statement is true or false:

(a) $R \subseteq M$

(b) $M \subset L$

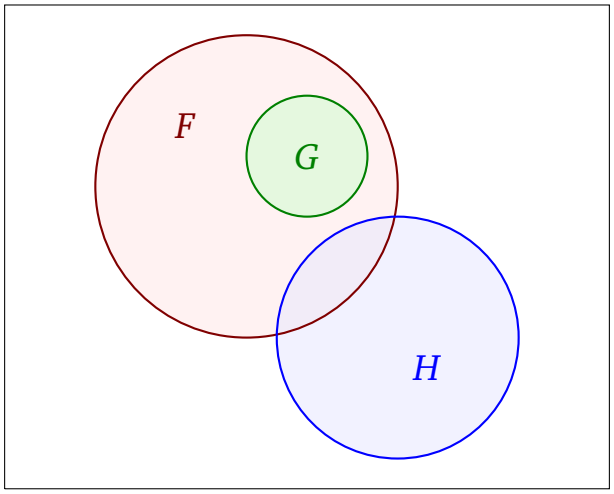
(c) $\{7\} \in M$

(d) $\{6, 8, 10\} \subseteq L$

(e) $P \subset R$

(f) $\{2\} \subset P$

7 (8 points)**.** The following diagram shows 3 sets of points on the plane, contained within the rectangular universal set U.



Determine whether the following statements are true or false:

(a) $G \subseteq F$

(b) $G \subset F$

(c) $H \subseteq F$

(d) $H \subset F$

(e) $G \subseteq U$

(f) $G \subseteq G$

(g) $G \subset G$

(h) $\varnothing \subset G$

8. Draw a diagram similar to the one above with sets $A \subset B \subset C \subset D$.

9. Use the $\subset$ relation to order the following ten sets, listed here without any particular order:

$W = \{x \mid x \text{ is a positive integer}\}$

$\mathbb{R}$

$M = \{3, 5, 7\}$

$\varnothing$

$D = \{x \mid x \text{ is a positive odd integer}\}$

$\mathbb{C}$

$P^* = \{x \mid x \text{ is a odd prime number}\}$

$\mathbb{Q}$

$N = \{x \mid x \text{ is a non-negative integer}\}$

$\mathbb{Z}$

10. A set X is called *transitive* when for every $A \in X$, it is also true that $A \subseteq X$. For each of the following sets, determine whether it is transitive. If a set is not transitive, prove it by stating a set A which provides a counterexample to the definition, that is: $A \in X$, but $A \nsubseteq X$.

(a) $\varnothing$

(b) $\{\varnothing\}$

(c) $\big\{\{\varnothing\}\big\}$

(d) $\big\{\varnothing, \{\varnothing\}\big\}$

(e) $\Big\{\varnothing, \{\varnothing\}, \big\{\{\varnothing\}\big\}\Big\}$

(f) The set $B = \{B\}$

(g) The set $C = \{C, D\}$, where $D = \{E\}$ and $E = \{D\}$, with $E \neq D$

Problems With Answers.

1. Use the roster notation to list the elements of the set of English letters in the word *Mississipi*.

2. Use the roster notation to list the days of the week whose names contain the letter u.

3. Use the roster notation to list months whose names end with the letter y.

4. Use a formal notation to describe the set of all people who traveled outside of the Solar system and came back to tell their story.

5. Use the roster notation to represent the set $V = \{v \mid v \text{ is an English vowel}\}$.

6. Use the roster notation to describe the set of English letters in this sentence.

7. Use the roster notation to describe the set of all countries whose names start with "Z".

Consider the sets defined below:

$$U = \{a, b, c, \ldots, x, y, z\} \qquad \text{(the universe is the set of all English letters)}$$
$$W = \{f, r, a, c, t, i, o, n\}$$
$$C = \{c, a, t\}$$
$$Y = \{a, y, e\}$$

Determine whether each statement is true or false:

8. $w \in W$

9. $a \in Y$

10. $z \in W$

11. $\{c\} \in C$

12. $C \in U$

13. $C \subseteq W$

14. $W \subseteq U$

15. $Y \subseteq W$

Define sets A, B, F, and M as follows:

$$\begin{aligned} A &= \{0, 3, 6, 9\} \\ B &= \{-1, -4, -7, -10\} \\ F &= \left\{k \,\middle|\, k \text{ is an integer multiple of } 5\right\} \\ M &= \left\{n \,\middle|\, n \text{ is a negative integer}\right\} \end{aligned}$$

Determine whether the following statements are true or false:

16. $3 \in A$

17. $7 \in B$

18. $0 \in F$

19. $-25 \in F$

20. $-7 \in M$

21. $0 \in M$

22. $\{10, 15\} \subseteq F$

23. $\{-1, -2\} \in M$

24. $\varnothing \in A$

25. $\varnothing \subseteq A$

26. $B \subseteq M$

27. $F \subseteq F$

Rewrite each set using the roster notation.

28. $\{x \mid x \text{ is a prime number and } 9 \leq x \leq 19\}$

29. $\{x/2 \mid x \text{ is a even integer and } 7 < x < 17\}$

30. $\{5-6x^2 \mid x \text{ is an odd integer and } -3 < x < 3\}$

31. $\{4x-7 \mid x \text{ is an integer and } -7 \leq x \leq -5\}$

32. $\{1+10^n \mid n \text{ is a positive integer and } n \leq 6\}$

33. $\left\{\frac{x}{x+2} \mid x \text{ is an integer and } 0 \leq x \leq 4\right\}$

Find an expression for the left side of the set-builder notation, so as to generate the given set.

34. Express the set $\{4.1, 5.1, 6.1, 7.1, 8.1, 9.1, \ldots\}$ as

$$\{ \qquad\qquad \mid x \text{ is a positive integer}\}$$

35. Express the set $\{0, -3, -6, -9, -12, -15, \ldots\}$ as

$$\{ \qquad\qquad \mid m \text{ is a non-negative integer}\}$$

36. Express the set $\{0.1, 0.01, 0.001, 0.0001, 0.00001, 0.000001, \ldots\}$ as

$$\{ \qquad\qquad \mid k \text{ is a positive integer}\}$$

37. Express the set $\{0, 1, 2, 3, 4, 5, 6, \ldots\}$ as

$$\{ \qquad\qquad \mid x \text{ is an integer}\}$$

ANSWERS.

1. $\{M, i, s, p\}$

3. {January, February, May, July}

5. $\{a, e, i, o, u\}$, though sometimes y is also included.

7. {Zambia, Zimbabwe}

9. true

11. false

13. true

15. false

17. false

19. true

21. false

23. false

25. true

27. true

29. $\{8, 10, 12, 14, 16\}$

31. $\{-35, -31, -27\}$

33. $\left\{0, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}\right\}$

35. $\left\{-3m \;\middle|\; m \text{ is a non-negative integer}\right\}$

37. $\left\{|x| \;\middle|\; x \in \mathbb{Z}\right\}$