

SET OPERATIONS

TEXT: 1.4, 1.5

LAST NAME	FIRST NAME	DATE
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1. For this activity, you will be divided into groups of 3 or 2. If you are in a pair, pretend that the choice (C) is not here.

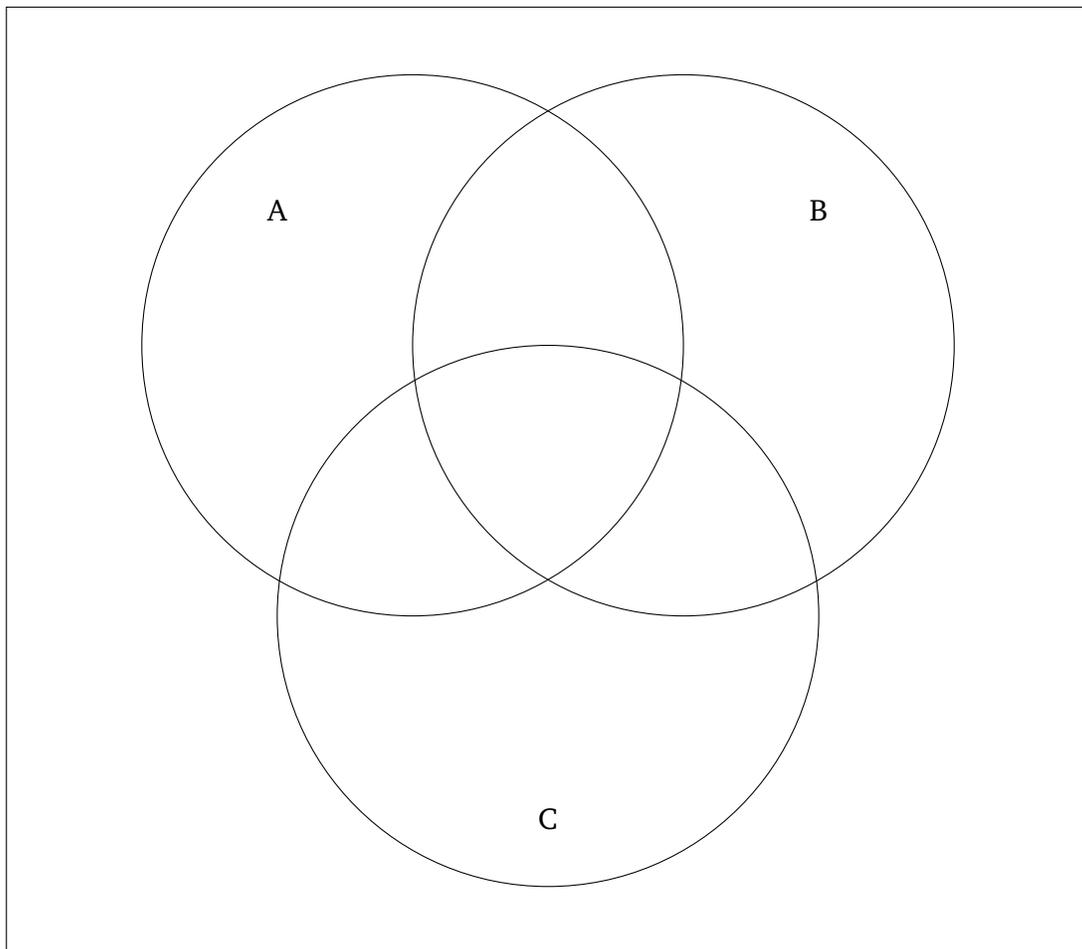
Introduce yourself to your group and write down your names:

A:

B:

C:

Talk with your group mates and find out what kind of interests they have. Write the interests in appropriate areas of the diagram below. The goal is to fill out all 8 areas of the diagram (4 areas if you are in a pair).



2 (7 points). The universe U and the sets within it are defined as follows:

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{2, 3, 5, 7, 9\}$$

$$B = \{0, 1, 5, 9\}$$

$$C = \{0, 1, 9\}$$

Use the roster notation or \emptyset to represent the following sets:

(a) $A \cup B$

(b) $A \cap B$

(c) $B - C$

(d) $C - B$

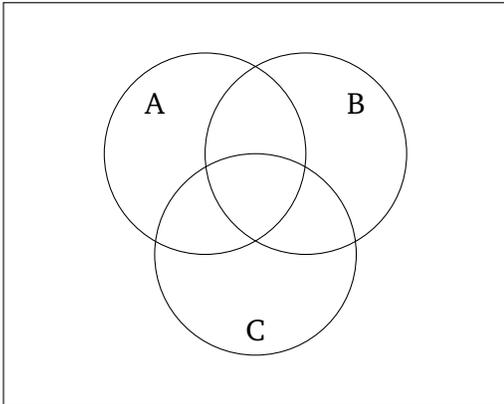
(e) $(A \cup C)'$

(f) $(A') \cap B$

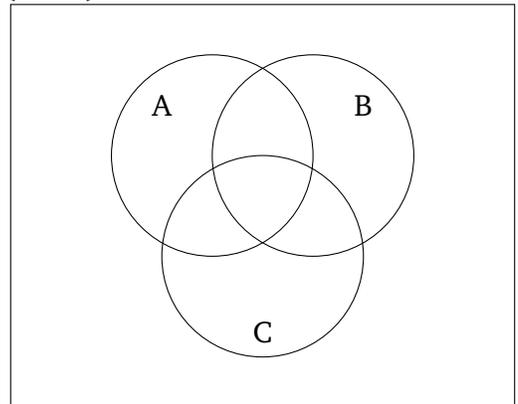
(g) $(A \cup B) - C$

3 (6 points). Use Venn diagrams to shade the following sets:

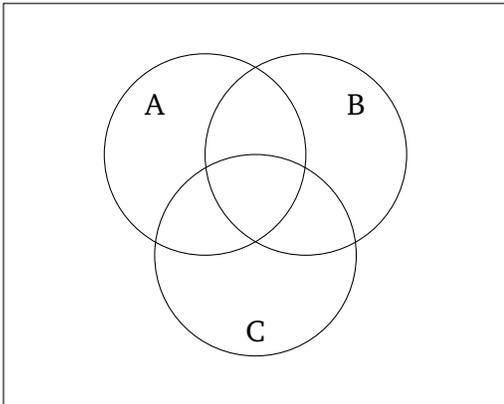
(a) $A \cap C$



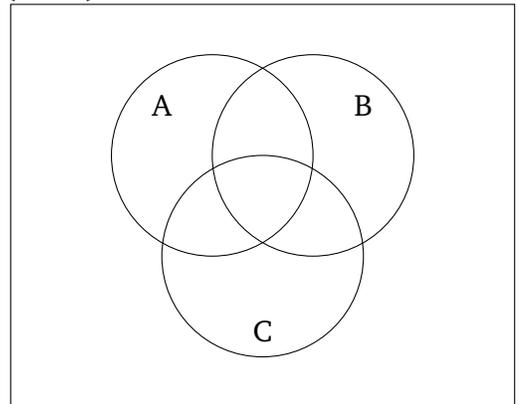
(d) $(A \cup B)'$



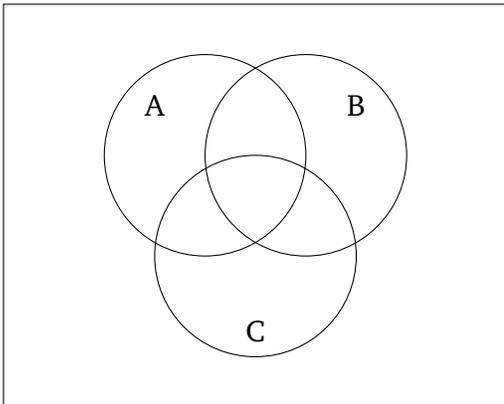
(b) $B \cup C$



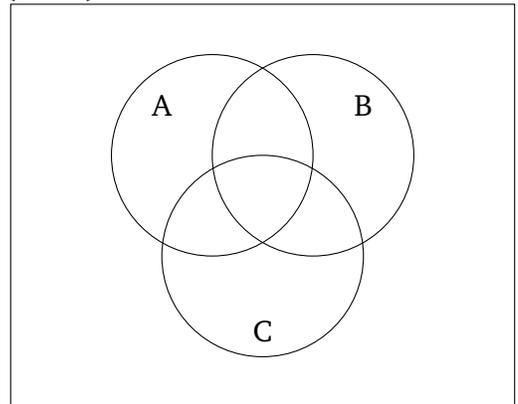
(e) $(A \cup B) \cap C$



(c) $B - C$

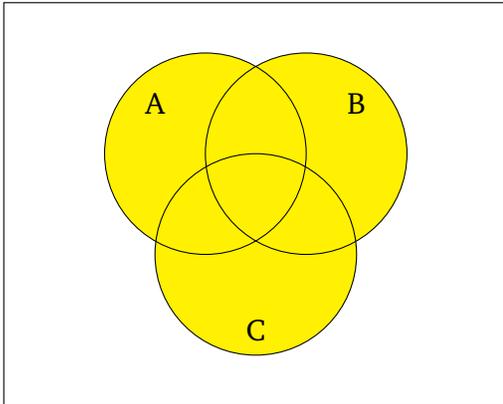


(f) $(A \cap B) \cap C$

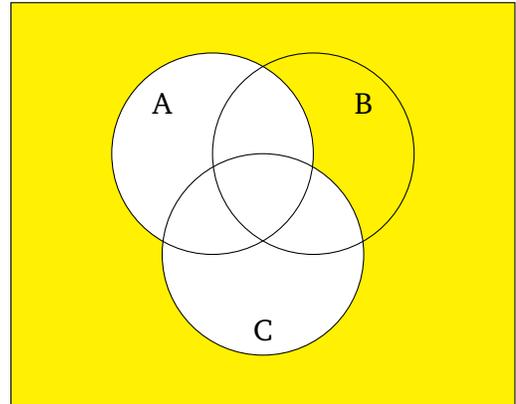


4 (6 points). For each of the Venn diagrams, write an algebraic expression for the shaded set:

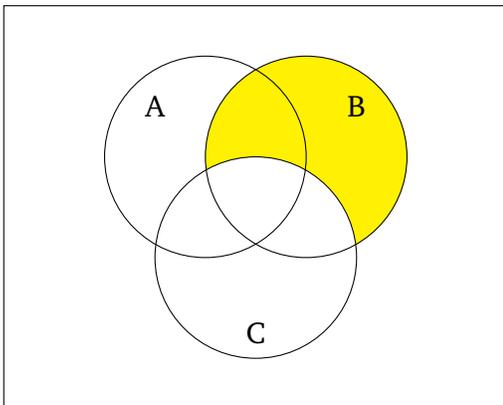
(a)



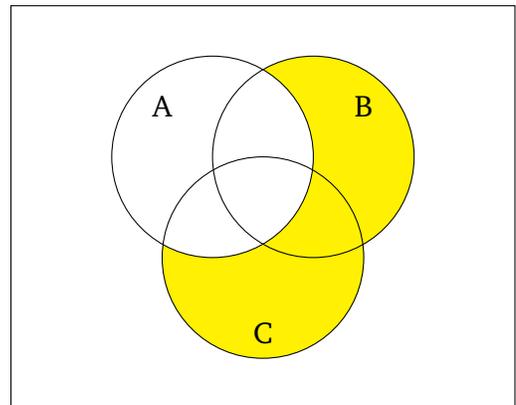
(d)



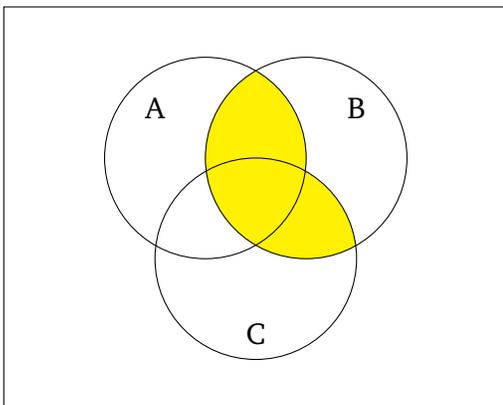
(b)



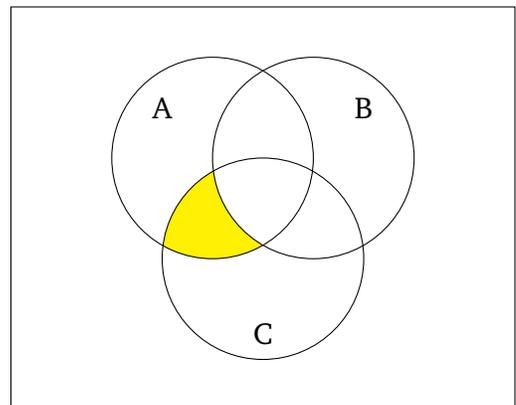
(e)



(c)



(f)



5. Draw a Venn diagram for four sets. To make it a true Venn diagram, it needs to have every possible intersection of these four sets represented by some shape/area. For an additional challenge, try to make it symmetric. Finally, shade the set representing

$$(A - B) \cup (B - C) \cup (C - D) \cup (D - A)$$

6 (3 points). Draw a diagram with a rectangle representing the universe of points within, and shapes inside the rectangle representing nonempty sets A , B , and C such that

- $A \cap B \neq \emptyset$
- $B \cap C \neq \emptyset$
- $A \cap C = \emptyset$

7 (3 points). Draw a diagram with a rectangle representing the universe of points within, and shapes inside the rectangle representing nonempty sets W , X , Y , and Z such that

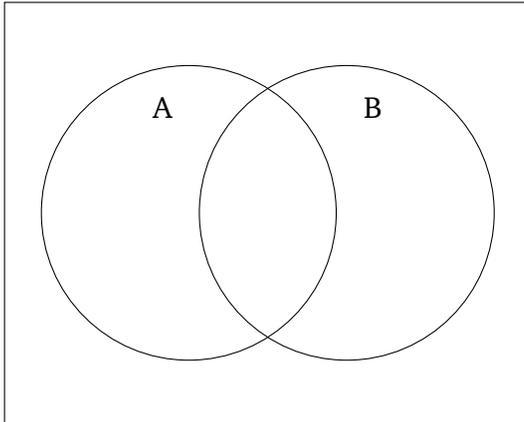
- $Y \subset W \cap X$
- $Z \subset (W \cup X)'$

8 (3 points). Draw a diagram with a rectangle representing the universe of points within, and shapes inside the rectangle representing nonempty sets K , L and M such that

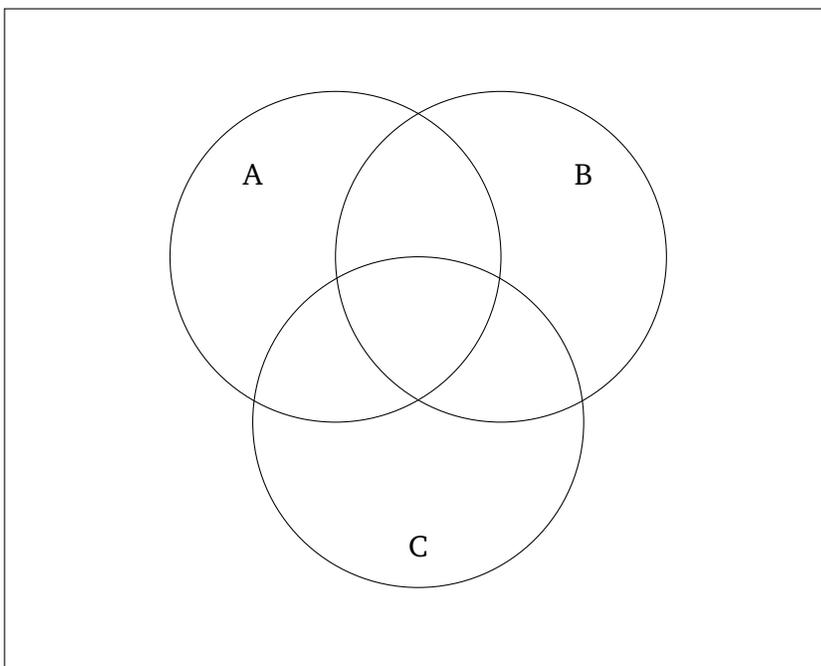
- $K \subset L$
- $M \cap K \neq \emptyset$
- $M \cap L' \neq \emptyset$
- $M \cap (L - K) = \emptyset$

9. Fill out a Venn diagram with elements placed into the appropriate areas. Each element should appear in your answer exactly once.

- (a) $U = \{1, 2, 3, 4, 5, 6, 7\}$,
 $A = \{2, 3, 4\}$,
 $B = \{3, 5, 6, 7\}$



- (b) $U = \{d, e, f, g, h, i, j, k\}$,
 $A = \{d, e, h, i, k\}$,
 $B = \{e, f, g, h, i\}$,
 $C = \{h, i, k\}$



PROBLEMS WITH ANSWERS.

Let U be the universal set, and let the other sets be as follows:

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 3, 5\}$$

$$C = \{4, 6\}$$

Find the following sets and state them using the roster notation.

1. $A \cup B$

2. $A \cup C$

3. $A \cap C$

4. $B \cap C$

5. A'

6. B'

Let $D = \{d, o, g, s\}$, $E = \{c, o, i, n, s\}$, and $F = \{d, o, c, k\}$. The universal set is the set of all English letters. Find the following sets and state them using the roster notation.

7. $E - D$

8. $D - F$

9. $(D \cap E) \cup F$

10. $D \cap (E \cup F)$

11. $D - (F \cap E)$

12. $F - (D \cup E)$

Draw the generic Venn diagrams showing sets A, B, C , with the following sets shaded.

13. $(B \cap A) \cup C$

15. $(B' \cap A') \cap C$

14. $(C \cup A)' \cap B$

16. $(A \cup C) \cup B$

Recall that given a set X , we write $\mathcal{P}(X)$ for the set of all subsets of X , or *power set* operation, for short. State the following sets using the roster notation.

17. $\mathcal{P}(\{a\})$

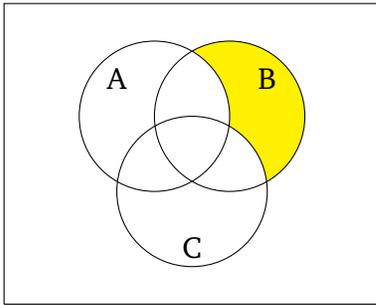
18. $\mathcal{P}(\{1, 2\})$

19. $\mathcal{P}(\emptyset)$

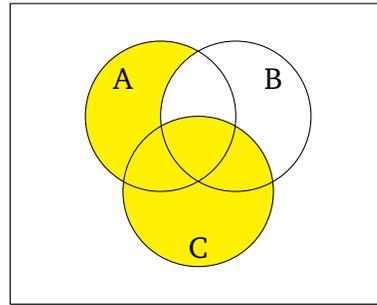
20. $\mathcal{P}(\{w, x, y, z\})$

Write an expression for the shaded region (many different correct answers are possible).

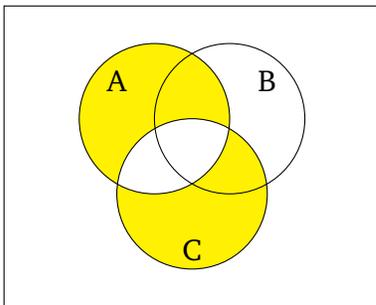
21.



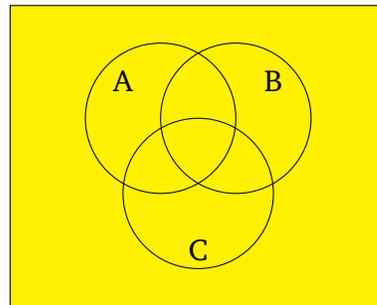
24.



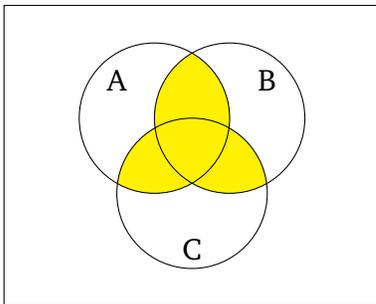
22.



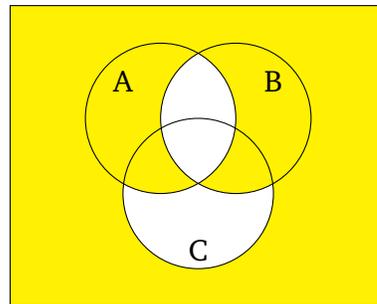
25.



23.



26.



Prove that the following laws hold for all sets by constructing a Venn diagram for each side of the given identities.

27. De Morgan's laws:

$$(P \cup Q)' = P' \cap Q'$$

$$(P \cap Q)' = P' \cup Q'$$

28. Distributivity laws:

$$(P \cup Q) \cap R = (P \cap R) \cup (Q \cap R)$$

$$(P \cap Q) \cup R = (P \cup R) \cap (Q \cup R)$$

ANSWERS.

1. $\{1, 2, 3, 4, 5\}$

3. $\{4\}$

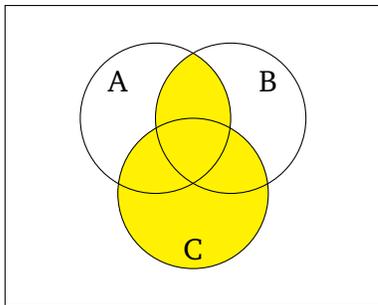
5. $\{0, 6, 7, 8, 9, 10\}$

7. $\{c, i, n\}$

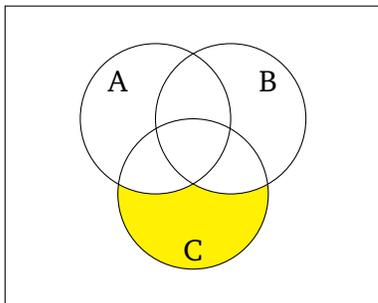
9. $\{d, o, c, k, s\}$

11. $\{d, g, s\}$

13.



15.



17. $\{\emptyset, \{a\}\}$

19. $\{\emptyset\}$

21. $(B - A) - C$

23. $(A \cap B) \cup (B \cap C) \cup (C \cap A)$

25. $A \cup (A')$