CARDINALITY

TEXT: 1.1

LAST NAME	FIRST NAME	DATE

1 (10 points)**.** Find cardinalities of the following sets:

(a) $\varnothing$

(b) $\{2, 3, 5, 7, 11, 13\}$

(c) The set of US states

(d) The set of months with 31 days

(e) $\{\varnothing, \{\varnothing\}\}$

(f) $\mathscr{P}(\varnothing)$

(g) $\mathscr{P}(\{\varnothing, \{\varnothing\}\})$

(h) $\mathscr{P}(\{2, 3, 5, 7\})$

(i) $\mathscr{P}(X)$, where $|X| = 12$

(j) The set of English letters which appear in the word "telemetry"

2. Find the cardinality of the set $D = \left\{ d \;\middle|\; d \text{ is the number of days in a calendar month} \right\}$.

3 (4 points)**.** Compare **cardinalities** of the given sets by inserting an appropriate sign:

$<$, $>$, or $=$

|real numbers between 0 and 1|	|real numbers between 0 and 100|
|rational numbers|	|irrational numbers|
|prime numbers|	|negative integers|
$\lvert\mathscr{P}(\{7, 8, 9\})\rvert$	$\lvert\{7, 8, 9\}\rvert$

4 (6 points)**.** Show that two sets have the same cardinality by thinking up a rule for pairing up their elements. State the rule by making paired lists, and/or by writing a formula relating the elements of two sets.

Example:

X, the set of positive integers
Y, the set of negative even integers

Solution:

$$X = \{\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots\}$$
$$Y = \{\ -2,\ -4,\ -6,\ -8,\ -10,\ -12,\ \ldots\}$$
$$y = -2x$$

(a)

A, the set of positive even integers
B, the set of negative odd integers

(b)

$M = \{7x + 3 \mid x \text{ is a positive integer}\}$
N, the set of negative integers

(c)

$\mathbb{W}$, the set of positive integers
X, the set of all positive integers written using only nines.

PROBLEMS WITH ANSWERS.

In the following exercises, determine the cardinality of each set, and which one is larger, or whether their cardinalities are the same.

1. $W =$ the set of days in a week, and $B =$ the set of months in a year.

2. $B =$ the set of letters used to spell the word "banana", and $T =$ the set of letters used to spell the word "totoro".

3. $X = \{1, 2, 3\}$, and $Y = \mathscr{P}(X)$.

4. $E =$ all positive even integers, and $D =$ all positive odd integers.

5. $N =$ all negative integers, and $T = \{10, 20, 30, 40, \ldots\}$ all positive integer multiples of ten.

6. $P =$ prime numbers between 2 and 100, and $\mathbb{Q} =$ all rational numbers.

7. $\mathbb{Z} =$ all integers, and $I =$ all rational numbers between 0 and 1.

8. $J =$ all irrational numbers, and $\mathbb{R} =$ all real numbers.

9. $I =$ all real numbers between -10 and 10, and $N =$ all negative rational numbers.

10. $\mathbb{R} =$ all real numbers, and $P = \mathscr{P}(\mathbb{R})$.

11. $S =$ the set of positive integer divisors of the number 60, and $E =$ the set of positive integer divisors of the number 80.

ANSWERS.

1. B is larger with $12 > 7$.

3. Y is larger with $8 > 3$.

5. The same cardinality $\aleph_0$.

7. The same cardinality $\aleph_0$.

9. I is larger with $\mathfrak{c} > \aleph_0$.

11. S is larger with $12 > 10$.