Logical Connectives

TEXT: 2.1, 2.2

Last Name	First Name	Date

1 (6 points)**.** Write each sentence in a symbolic form. Use m, a, j, r, d, as defined below.

$$m = \text{“Petro will graduate in May”}$$
$$a = \text{“Petro will graduate in August”}$$
$$j = \text{“Petro can find a summer job”}$$
$$r = \text{“Petro can rent a studio apartment”}$$
$$d = \text{“Petro will adopt a dog”}$$

(a) Petro can find a summer job and rent a studio apartment.

(b) If Petro will not graduate in May, then he will graduate in August.

(c) Petro will graduate in May, or else he can find a summer job and graduate in August.

(d) If Petro cannot find a summer job, then he cannot rent a studio apartment and he will not graduate in May.

(e) Petro will adopt a dog if he rents a studio apartment.

(f) Petro will rent a studio apartment and adopt a dog iff he can find a summer job.

2 (4 points). Write each symbolic statement as an English sentence. Use p, e, s, i, t as defined below.

$$p = \text{"the number is prime"}$$
$$e = \text{"the number is even"}$$
$$s = \text{"the number is less than 6"}$$
$$i = \text{"the number is imaginary"}$$
$$t = \text{"the number is 2"}$$

(a) $p \lor e$

(b) $e \land s$

(c) $t \rightarrow e$

(d) $i \rightarrow (\sim p)$

3. Let $b =$ "I buy one item" and $g =$ "I get one free item".
How would you translate the ad shown above into a formal statement?

4. Can you think of a sentence that is always true, no matter who says it or in what circumstances? If so, can you write it down formally?

5. Can you think of a sentence that is always false, no matter who says it or in what circumstances? If so, can you write it down formally?

6 (8 points). Write the negation of each statement by switching the quantifier:

(a) All sheep are white.

(b) Some presidents are female.

(c) No one likes broccoli.

(d) Every cat believes it owns the house.

(e) There exists a horse with a horn.

Write the negation of each statement by switching the quantifier(s):

(f) Some dogs believe they are humans.

(g) All cars do not pollute.

(h) None of these statements make any sense.

(i) There is a mirror in the Room Of Requirement such that for anyone who looks into it, the mirror shows something appealing.

(j) For any number x and for any number y there exists a number z such that $x < z < y$.

7 (5 points). Write each symbolic statement as an English sentence. Use M, I, R, C, P as defined below.

$$
\begin{aligned}
M &= \text{``The book is a mystery novel.''} \\
I &= \text{``The book has illustrations.''} \\
R &= \text{``I am going to read the book.''} \\
C &= \text{``The book ends in a cliffhanger.''} \\
P &= \text{``The book is written in poetry.''}
\end{aligned}
$$

(a) $(\sim P) \longleftrightarrow (\sim I)$

(b) $(I \rightarrow (\sim P)) \wedge ((\sim I) \rightarrow C)$

(c) $(P \vee C) \rightarrow (I \vee (\sim R))$

(d) $M \rightarrow (I \rightarrow R)$

(e) $(M \rightarrow I) \rightarrow R$

PROBLEMS WITH ANSWERS.

Rewrite each sentence in the symbolic form. Represent each simple statement with a letter provided in the brackets.

1. If today is Sunday [s], then tomorrow is Monday [m].

2. I went to the hardware store [h] and the post office [p].

3. A triangle is equilateral [l] if and only if it is equiangular [a].

4. The number is divisible by nine [t] or it is divisible by seven [s].

5. Grover can bark [b] only if Grover is a dog [d].

6. The sky is clear today [c], but the wind is strong [w].

7. If the dog is asleep [a], then he is not chasing squirrels [s].

8. My computer does not crash [c], and it is powered by a hamster in a wheel [h].

9. If I am late [l], then the traffic was bad [t] or the gremlins hid my keys [k].

10. I am not of the East [e], nor of the West [w], nor of the land [s], nor of the sea. [s]

Rewrite the following symbolic statements in English, using the following variable assignments:

s: Leon Bridges is a singer

w: Leon Bridges is a songwriter

a: Leon Bridges is an actor

g: Leon Bridges plays the guitar

v: Leon Bridges makes music videos

11. $(s \lor w) \land g$

12. $s \to ((\sim a) \land g)$

13. $(a \land s) \leftrightarrow v$

14. $(\sim v) \to (\sim a)$

15. $(\sim g) \to (s \lor w)$

16. $(g \land (\sim w)) \to v$

Negate each of the following statements by switching the quantifiers:

17. Some socks got lost in the laundry.

18. Every bird likes to dance to a good tune.

19. All superheroes wear their underwear on the outside.

20. There exists a person who has never told a dad joke.

21. Some pleasures do not involve any guilt.

Negate each of the following statements by switching the quantifiers:

22. None of the students ever procrastinate on their homework.

23. For every nail there exists a hammer that can drive it in.

24. On every pizza there is at least one topping that someone claims to hate.

25. For some musicians, every song of theirs is a hit.

26. For every plant there is a season when it doesn't grow.

Vitali Klitschko is a Ukranian professional boxer, who also defended a PhD in sports science, and served as the mayor of Kiev. Write each of the following sentences in symbolic form, using the following variable assignments:

$$b: \quad \text{Vitali is a boxer}$$
$$p: \quad \text{Vitali is a politician}$$
$$r: \quad \text{Vitali wants to run for president}$$
$$v: \quad \text{Vitali is very popular}$$
$$l: \quad \text{Vitali has lost his title}$$

27. Vitali is a boxer and a politician, and he has not lost his title.

28. Vitali wants to run for president only if he is both a politician and is also very popular.

29. If Vitali does not want to run for president or has lost his title, then he is not very popular.

30. Vitali is either a boxer or not a boxer, and he is not a politician.

ANSWERS.

1. $s \rightarrow m$

3. $l \leftrightarrow a$

5. $b \rightarrow d$

7. $a \rightarrow (\sim s)$

9. $l \rightarrow (t \vee k)$

11. Leon Bridges is either a singer or a songwriter, and he plays guitar.

13. Leon Bridges is an actor and a singer if and only if he makes music videos.

15. If Leon Bridges does not play the guitar, then he is either a singer or a songwriter.

17. None of the socks got lost in the laundry.

19. Some superheroes do not wear their underwear on the outside.

21. Every pleasure involves some guilt.

23. There exists a nail such that no hammer can drive it in.

25. For every musician there is at least one song of theirs that is not a hit.

27. $(b \wedge p) \wedge (\sim l)$

29. $((\sim r) \vee l) \rightarrow (\sim v)$