EQUIVALENT STATEMENTS 2.4, 2.5, 2.6

LAST NAME	FIRST NAME	DATE

1 (6 points)**.** For each of the following statements, construct side-by-side truth tables, fill out answer columns completely, and box them. Also, for each pair of statements, determine whether they are equivalent.

(a)

$$x \longleftrightarrow (\sim y)$$

$$x \wedge (\sim y)$$

Equivalent Not equivalent

(b)

$$(P \vee Q) \rightarrow R$$

$$(P \rightarrow R) \wedge (Q \rightarrow R)$$

Equivalent Not equivalent

2 (6 points)**.** Consider the following statement:

If I sleep in today, then I will be late for class.

Write down the following statements:

(a) converse:

(b) inverse:

(c) contrapositive:

3 (6 points)**.** Consider the following statement:

If slopes of two lines are not equal, then the two lines intersect at a point.

Write down the following statements:

(a) converse:

(b) inverse:

(c) contrapositive:

4 (6 points)**.** Determine whether these statements are equivalent, and how they relate to each other (are they mutual converses, inverses, or contrapositives?):

(a)
- If I am not a US citizen, then I cannot vote in federal elections.
- If I am a US citizen, then I can vote in federal elections.

(b)
- If I am drunk, then I am not allowed to drive.
- If I am allowed to drive, then I am not drunk.

(c)
- If the water is wet, then water is not wet.
- If the water is not wet, then water is wet.

5. Rewrite the given statement as a disjunction, and its negation as a conjunction:

If something is not broken, then we should not fix it.

As a disjunction:

Negation as a conjunction:

6 (6 points)**.** For each of the following statements, construct side-by-side truth tables, fill out answer columns completely, and box them. Also, for each pair of statements, determine whether they are equivalent.

(a)

$$(x \wedge y) \vee \big((\sim x) \wedge (\sim y)\big)$$

$$x \longleftrightarrow y$$

Equivalent Not equivalent

(b)

$$(A \longleftrightarrow B) \vee C$$

$$(A \longleftrightarrow C) \vee (B \longleftrightarrow C)$$

Equivalent Not equivalent

7. Define the connective $A \oplus B$, known as *exclusive or* or *XOR*, by the following truth table:

A	B	$A \oplus B$
T	T	F
T	F	T
F	T	T
F	F	F

Find a statement using the five connectives defined so far, which is equivalent to $A \oplus B$.

8. Define the connective $A\#B$, known as *NAND*, by the following truth table:

A	B	$A\#B$
T	T	F
T	F	T
F	T	T
F	F	T

Find a statement using the five connectives defined so far, which is equivalent to $A\#B$.

Construct a truth table for the following statement:

$$\big(x \oplus (y\#x)\big)\#(y \oplus x)$$

9. Use # and only # to express each of the five connectives we've defined so far, as well as the *exclusive or* $\oplus$. For example, (and you should check this via a truth table) the negation $\sim A$ can be expressed as $A\#A$.

Problems With Answers.

Use truth tables to determine whether the two statements are equivalent.

1. $p \vee \big(q \wedge (\sim p)\big)$ versus $q \vee p$

Equivalent Not equivalent

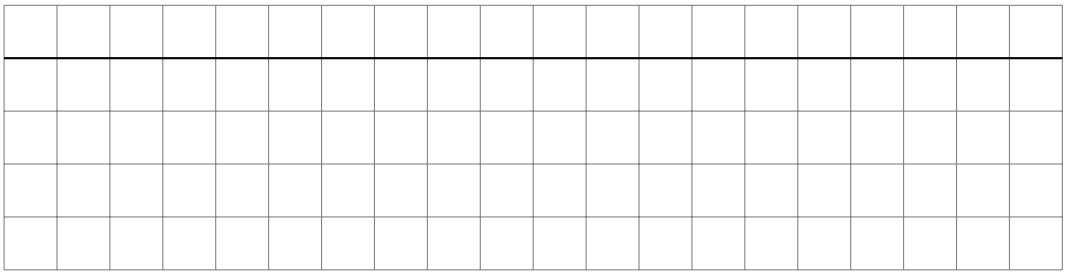
2. P versus $\sim P \rightarrow (Q \wedge (\sim Q))$

Equivalent Not equivalent

3. $\sim (a \vee b)$ versus $(\sim a) \vee (\sim b)$

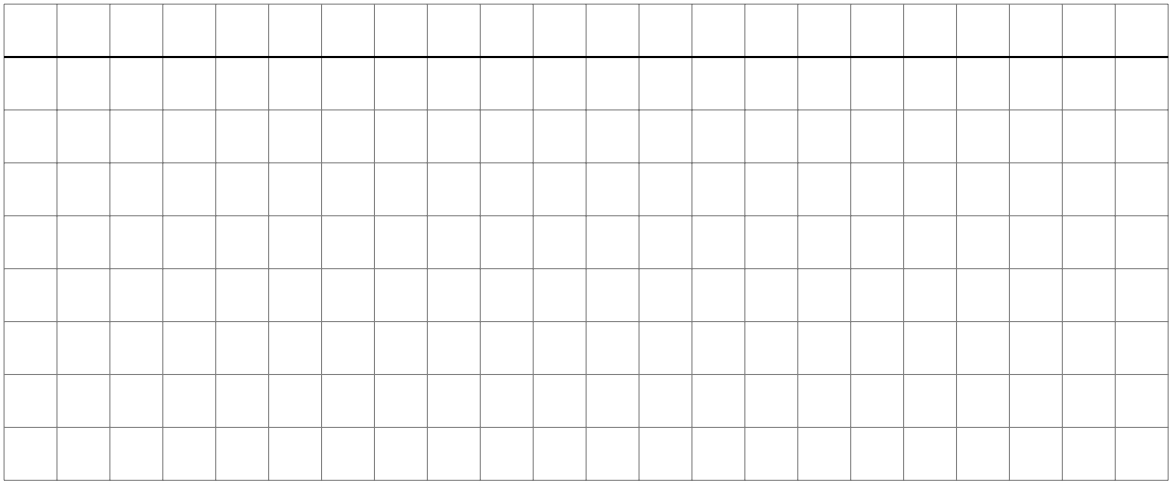
Equivalent Not equivalent

4. $\sim(a \wedge (\sim b))$ versus $(\sim a) \vee b$



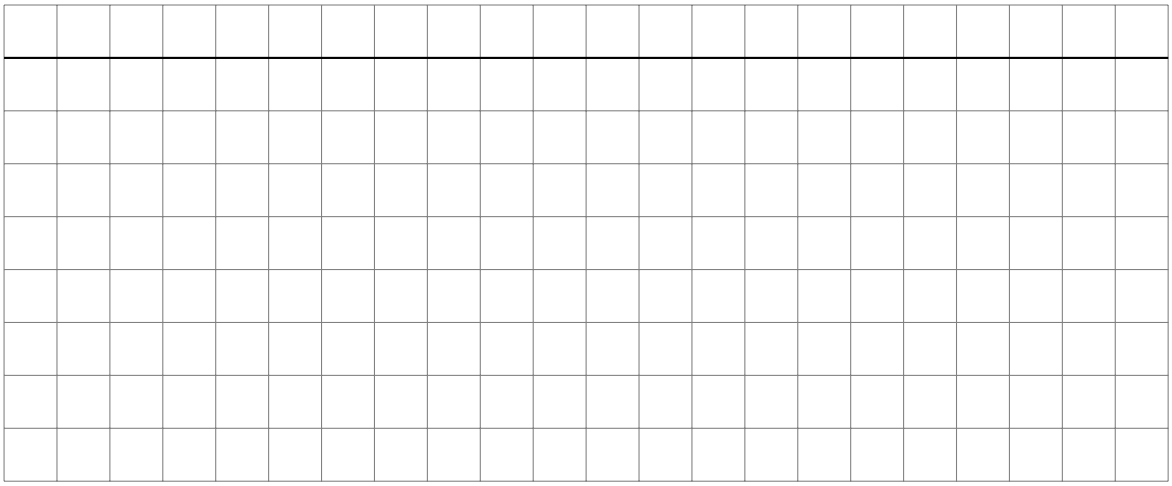
Equivalent Not equivalent

5. $\big((x \to y) \wedge (y \to z)\big) \wedge (z \to x)$ versus $(y \leftrightarrow z) \wedge (y \leftrightarrow x)$



Equivalent Not equivalent

6. $x \to (y \vee z)$ versus $(x \to y) \vee (x \to z)$



Equivalent Not equivalent

7. Write the converse, the inverse, and the contrapositive for the statement:

If you are under age 17, then you cannot attend this movie.

Converse:

Inverse:

Contrapositive:

8. Write the converse, the inverse, and the contrapositive for the statement:

Shaneese will go to work only if she doesn't have a fever.

Converse:

Inverse:

Contrapositive:

For each of the following conditionals,
(a) rewrite it as a disjunction and
(b) write down its negation as a conjunction.

9. *If Luke faces Vader, then Obi-Wan cannot interfere*
As a disjunction:

Negation as a conjunction:

10. *If you don't look both ways before crossing the street, then you might get hit by a car.*
As a disjunction:

Negation as a conjunction:

11. *If you don't eat your meat, you can't have any pudding!*

-Pink Floyd

As a disjunction:

Negation as a conjunction:

12. *Math is fun only if it's completely and totally useless.*
As a disjunction:

Negation as a conjunction:

13. *If you think you understand quantum mechanics, then you don't understand quantum mechanics.*

-Attributed to Richard Feynman

As a disjunction:

Negation as a conjunction:

14. Assume that the statement

If you swear, then you will get your mouth washed out with soap

is true. Which of the following statements must also be true?

(a) If you don't swear, then you won't get your mouth washed out with soap.
(b) If you don't get your mouth washed out with soap, then you didn't swear.
(c) If you get your mouth washed out with soap, then you swore.

ANSWERS.

1. Equivalent:

p	q	$p \vee \big(q \wedge (\sim p)\big)$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

3. Not equivalent:

a	b	$\sim (a \vee b)$	$(\sim a) \vee (\sim b)$
T	T	F	F
T	F	F	T
F	T	F	T
F	F	T	T

5. Equivalent:

x	y	z	$\big((x \to y) \wedge (y \to z)\big) \wedge (z \to x)$	$(y \leftrightarrow z) \wedge (y \leftrightarrow x)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	T	T

7.
Converse: *If you cannot attend this movie, then you are under age 17.*
Inverse: *If you are not under age 17, then you can attend this movie.*
Contrapositive: *If you can attend this movie, then you are not under age 17.*

9.
Disjunctive form: *Either Luke does not face Vader, or Obi-Wan cannot interfere.*
Negation as a conjunction: *Luke faces Vader, and Obi-Wan can interfere.*

11.
Disjunctive form: *Either you eat your meat, or you can't have any pudding!*
Negation as a conjunction: *You don't eat your meat, but you can have some pudding!*

13.
Disjunctive form: *Either you don't think you understand quantum mechanics, or you don't understand quantum mechanics.*
Negation as a conjunction: *You think you understand quantum mechanics, and you do understand quantum mechanics.*