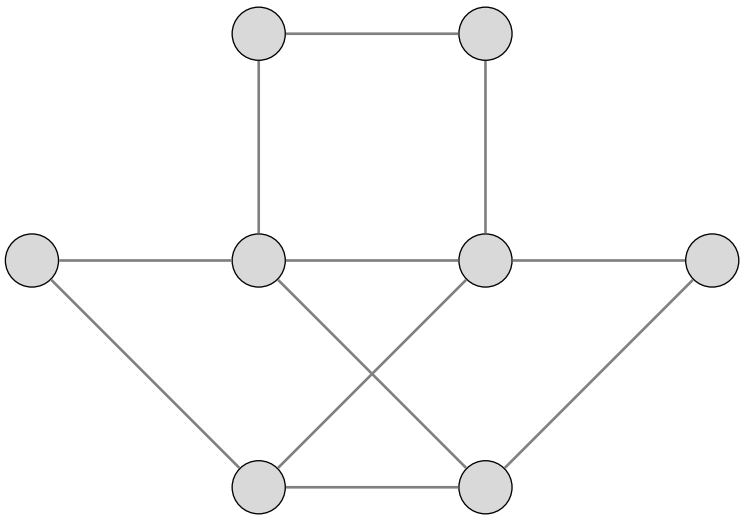
COLORING

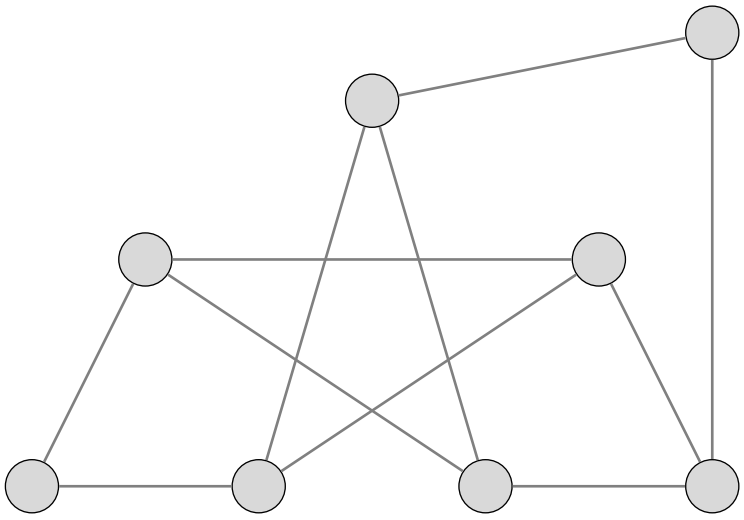
LAST NAME	FIRST NAME	DATE

1 (8 points)**.** For each of the following graphs either find a coloring using only 2 colors, or prove that more than 2 colors are needed by finding and highlighting a cycle of odd length.

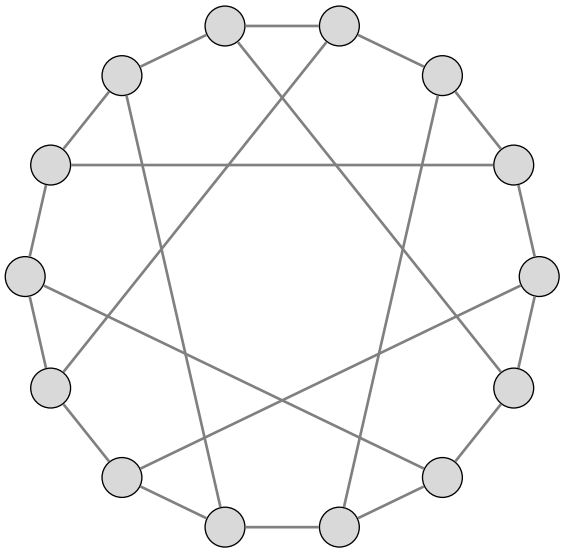
(a)



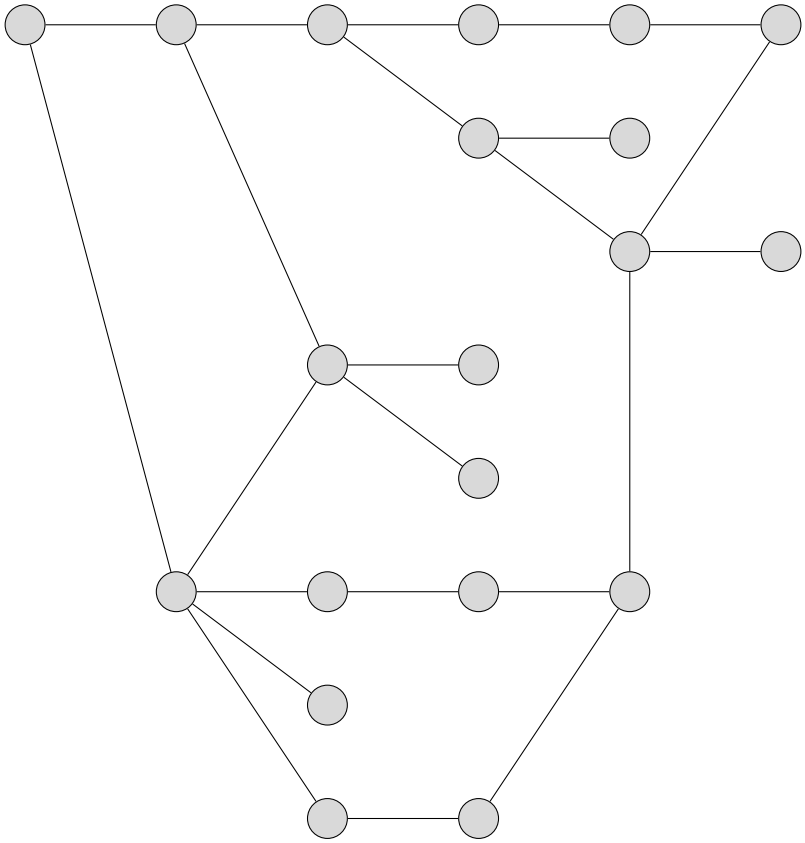
(b)



(c) Heawood graph

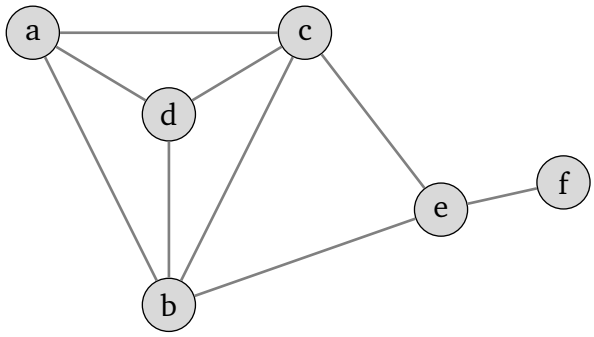


(d)

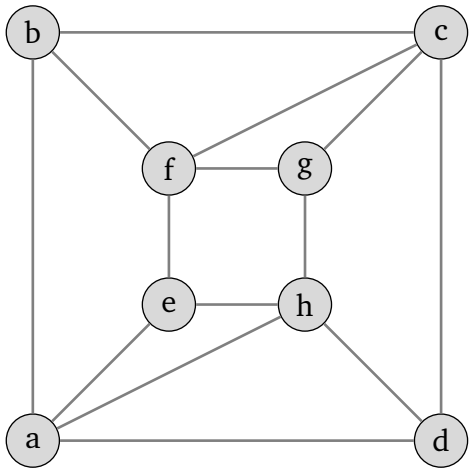


2 (3 points)**.** For each shown graph, draw a possible map. Color the graph or the map with as few colors as possible, so that no two ajacent countries share a color. State the chromatic number for each graph.

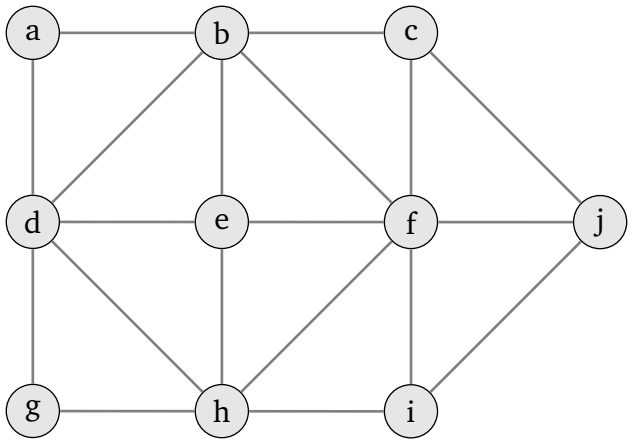
(a) $\chi =$



(b) $\chi =$



(c) $\chi =$



3 (4 points)**.** Represent the map by a graph, find the chromatic number of the graph, and then find an optimal graph coloring.

4. Martin Gardner played an April Fool's joke in April 1st, 1975 edition of Scientific American by asserting that the map of 110 regions illustrated below requires five colors and constitutes a counterexample to the four-color theorem. Gardner wrote:

"William McGregor, a graph theorist of Wappingers Falls, N.Y., constructed a map of 110 regions that cannot be colored with fewer than five colors. McGregor's technical report will appear in 1978 in the Journal of Combinatorial Theory, Series B."

William McGregor is a real mathematician who created the map and gave Gardner permission to use it as an April Fool's prank. Try to color the McGregor map with as few colors as possible.