Practice Test 2. Set Theory

Math 300

Last Name	First Name	Date

This assignment is closed books. One 2-sided US Letter sheet of notes is OK.

All your scratch work will be collected with the test and discarded.

All electronic devices besides approved calculators and computers running approved software are prohibited.

Fully justify your answers and show all work
in order to maximize your partial credit.

1 (6 points)**.** Use the roster notation to represent the following sets.

(a) $\{x \mid 0 < x < 10 \text{ and } x \text{ is an integer multiple of } 3\}$

(b) $\{2^x \mid x \text{ is a non-negative integer and } x \leq 3\}$

(c) $\mathscr{P}(\{a, b\})$, that is, the set of all subsets of $\{a, b\}$

2 (4 points)**.** Find the **cardinalities** of the following sets:

(a) $\{x \mid x \text{ is a calendar day in December}\}$

(b) $\mathscr{P}(\{0, 1, 10, 100, 1000\})$

3 (12 points)**.** Consider the following sets:

$$\begin{aligned} A &= \{x \mid x \text{ is a positive integer}\} \\ B &= \{-1, 0, 1\} \\ C &= \{1, 2, 3\} \end{aligned}$$

True or false?

(a) $17 \in A$

(b) $\varnothing \in A$

(c) $\varnothing \subseteq A$

(d) $A \subseteq A$

(e) $B \subseteq A$

(f) $C \subseteq A$

4 (14 points)**.** Let the sets be defined as follows:

$$\begin{aligned} U &= \{1,2,3,4,5,6,7,8,9,10\} \\ A &= \{1,2,3,4,5,6\} \\ B &= \{5,6,7,8,9,10\} \\ C &= \{4,5,6\} \end{aligned}$$

True or false?

(a) $C \in A$

(b) $C \subseteq A$

(c) $C \subset A$

(d) $A = B$

(e) $|A| = |B|$

(f) $U \subset U$

Use the roster notation (or $\varnothing$) to represent the following sets:

(g) $A - C$

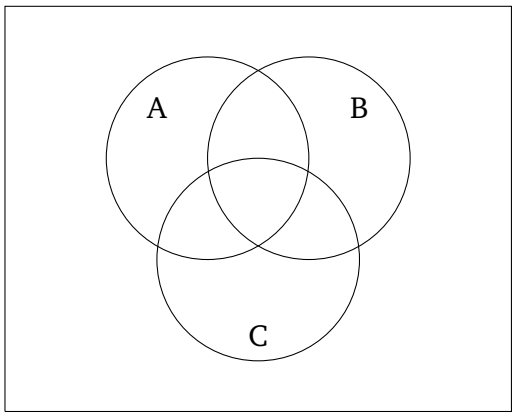
(h) $C - A$

(i) $(A \cup C)'$

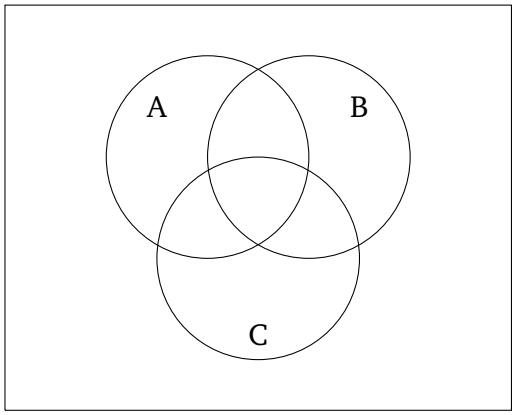
(j) $(A \cap B) \cap C$

5 (12 points)**.** Use Venn diagrams to shade the following sets:

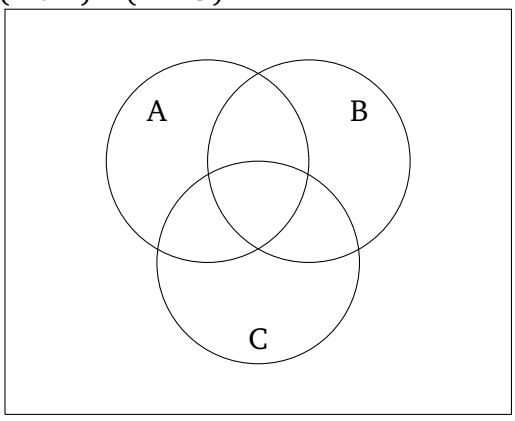
(a) $A \cup B$



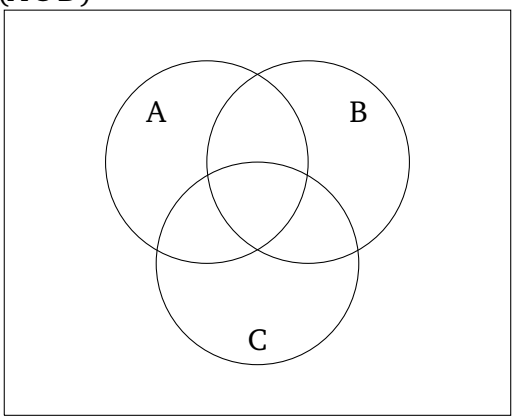
(b) $B \cap C$



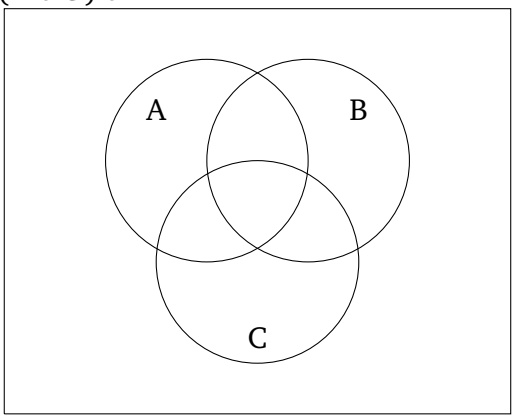
(c) $(A \cup B) - (B \cap C)$



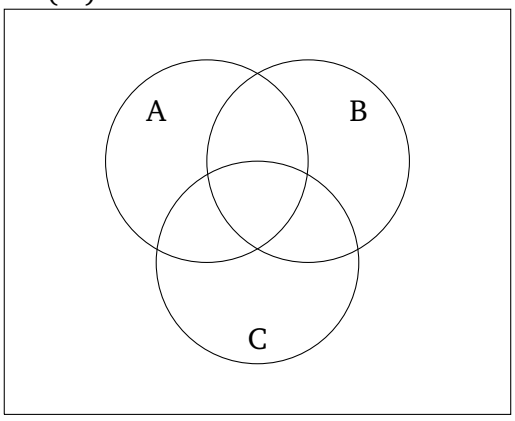
(d) $(A \cup B)'$



(e) $(B \cup C) \cup A$

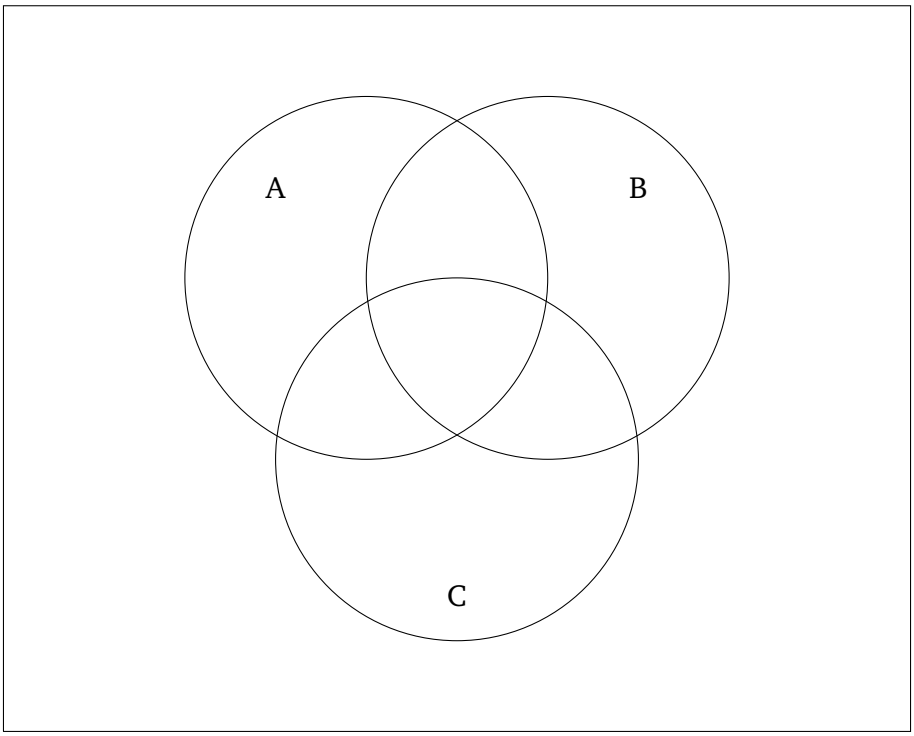


(f) $A \cap (A')$



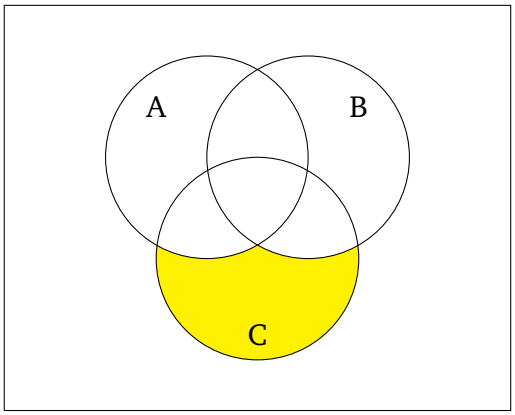
6 (8 points). Complete a Venn diagram by placing elements into appropriate areas:

$$\begin{aligned} U &= \{1,2,3,4,5,6,7\} \\ A &= \{1,3,5,7\} \\ B &= \{2,4,6\} \\ C &= \{3,4,5,6\} \end{aligned}$$

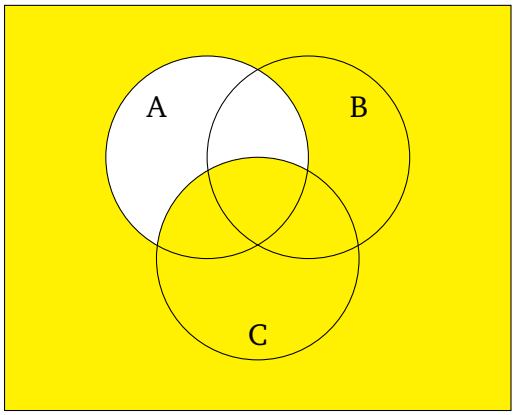


7 (4 points). For each diagram shown, write an algebraic expression for the shaded set:

(a)



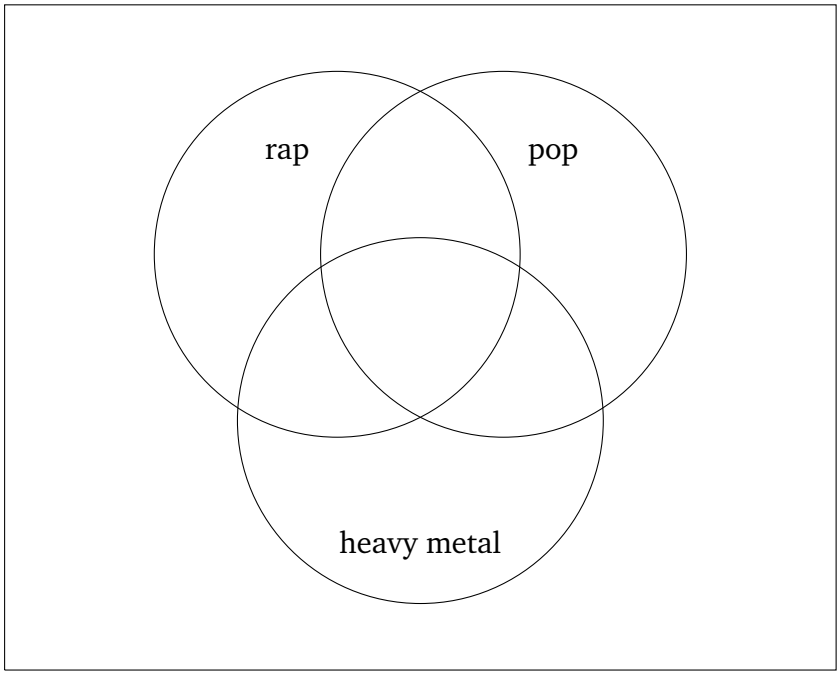
(b)



8 (8 points)**.** A music teacher has surveyed 495 students to find out which genres of music they like. The results of the survey are listed below.

320 students like rap
395 students like pop
295 students like heavy metal
280 students like both rap and pop
190 students like both rap and heavy metal
245 students like both pop and heavy metal
160 students like all three genres

(a) Fill out the diagram below by stating the cardinality for each of the 8 mutually disjoint subsets of the universe.



(b) How many students only like pop?

(c) How many students only like one of the three genres?

(d) How many students like rap and heavy metal, but not pop?

9 (4 points)**.** Compare **cardinalities** by inserting an appropriate sign between the descriptions of given sets: $<$, $>$, or $=$.

|integers|	|positive even integers|
|prime numbers between 0 and 10|	|odd integers between 0 and 10|
|real numbers|	|rational numbers|
|rational numbers|	|negative integers|

10 (6 points)**.** Show that two sets have the same cardinality by thinking up a rule for pairing up their elements. State the rule by making paired lists, and/or by writing a formula relating the elements of two sets.

Example:

X, the set of positive integers
Y, the set of negative even integers

Solution:

$$X = \{1, 2, 3, 4, 5, 6, \ldots\}$$
$$Y = \{-2, -4, -6, -8, -10, -12, \ldots\}$$
$$y = -2x$$

(a)

H, the set of negative integers
G, the set of positive integers greater than 5

(b)

$\mathbb{Z}$, the set of integers
T, the set of all integer powers of 10, $T = \left\{10^k \;\middle|\; k \text{ is an integer}\right\}$