Practice Test 3. Logic

Math 300

Last Name	First Name	Date

This assignment is closed books. One 2-sided US Letter sheet of notes is OK.

All your scratch work will be collected with the test and discarded.

All electronic devices besides approved calculators and computers running approved software are prohibited.

Fully justify your answers and show all work
in order to maximize your partial credit.

1 (10 points)**.** Write each sentence in a symbolic form. Use p, q, r, s, and t as defined below.

$$\begin{aligned} p &= \text{“the coffee is hot”} \\ q &= \text{“the lid is loose”} \\ r &= \text{“the burns are severe”} \\ s &= \text{“the customer is driving”} \\ t &= \text{“the customer is careful”} \end{aligned}$$

(a) The coffee is hot and the customer is driving.

(b) Either the coffee is hot or the coffee is not hot.

(c) If the coffee is hot and the lid is loose, then the customer is not careful.

(d) The customer is careful if and only if the following is true: the lid is not loose or the customer is not driving.

(e) Either the customer is careful and the lid is not loose, or the coffee is hot and the burns are severe.

2 (6 points)**.** Write each symbolic statement as an English sentence. Use p, r, c, d, and z as defined below.

$$\begin{aligned} p &= \text{“the solution is positive”} \\ r &= \text{“there is a real solution”} \\ c &= \text{“solutions are complex”} \\ d &= \text{“discriminant is negative”} \\ z &= \text{“discriminant is zero”} \end{aligned}$$

(a) $c \leftrightarrow (\sim r)$

(b) $r \rightarrow (p \wedge z)$

(c) $(z \vee (\sim d)) \rightarrow r$

3 (6 points)**.** Construct truth tables for the given statements and determine whether they are equivalent.

$$x \to (x \to x)$$

$$(x \to x) \to x$$

4 (6 points)**.** Construct a truth table to determine whether the following statement is a contradiction, a tautology, or neither:

$$\big((a \leftrightarrow c) \vee b\big) \wedge \big(a \to (\sim b)\big)$$

5 (6 points)**.** Consider the following statement:

If Fernando is driving a car, then he does not answer his phone.

Write down the following statements:

(a) converse:

(b) inverse:

(c) contrapositive:

6 (4 points)**.** Write a contrapositive for each statement:

(a) If I don't fix a typo in the Wikipedia article, then someone else has to do it.

(b) If a book does not have pictures, then it is not worth reading.

7 (8 points)**.** Write the negation of each statement by switching the quantifier.

(a) There exists an Earth-like planet outside of our Solar system.

(b) Every morning the Sun rises in the East.

(c) None of my siblings are younger than me.

(d) Some of the emails I get are not addressed to me.

8 (2 points)**.** How many rows will a truth table for this statement have?

$$\Big((a \to b) \vee (a \wedge c \wedge f)\Big) \leftrightarrow \Big((\sim d \vee e) \wedge f\Big)$$

9 (2 points)**.** Find the truth value of the given statement, if $p =$ true, $q =$ true, and $r =$ false:

$$\Big((p \vee r) \leftrightarrow \big((\sim r) \wedge (\sim q)\big)\Big) \rightarrow (\sim p)$$

10 (4 points)**.** Construct a truth table to determine whether the following statement is a contradiction, a tautology, or neither:

$$(a \wedge b) \vee (a \rightarrow (\sim b))$$

11 (4 points)**.** Construct a truth table to determine whether the following statement is a contradiction, a tautology, or neither:

$$\sim (x \vee (x \rightarrow y))$$

12 (8 points)**.** Construct a truth table to determine whether the following argument is valid:

$$\begin{array}{l} z \\ x \rightarrow (\sim z) \\ (\sim x) \rightarrow y \\ \hline \therefore x \wedge y \end{array}$$

13 (8 points)**.** Construct a truth table to determine whether the following argument is valid:

$$\begin{array}{l} a \\ b \vee (\sim c) \\ a \rightarrow (\sim b) \\ \hline \therefore \sim c \end{array}$$