Practice Test 4. Graphs

Math 300

Last Name	First Name	Date

This assignment is closed books. One 2-sided US Letter sheet of notes is OK.

All your scratch work will be collected with the test and discarded.

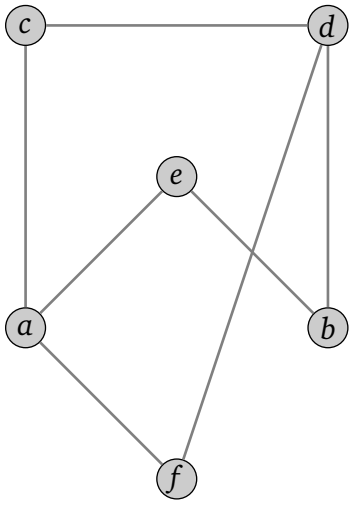
All electronic devices besides approved calculators and computers running approved software are prohibited.

Fully justify your answers and show all work
in order to maximize your partial credit.

1 (5 points)**.** Draw a graph defined by the adjacency matrix:

	A	*B*	*C*	*D*	*E*	*F*
A		*X*	*X*			
B	*X*		*X*			
C	*X*	*X*		*X*	*X*	*X*
D			*X*			
E			*X*			*X*
F			*X*		*X*	

2 (5 points)**.** Fill out the adjacency matrix for the shown graph:



	a	*b*	*c*	*d*	*e*	*f*
a						
b						
c						
d						
e						
f						

3 (15 points)**.** Draw the following graphs.

(a) The complete graph K_3

(b) The complete graph K_6

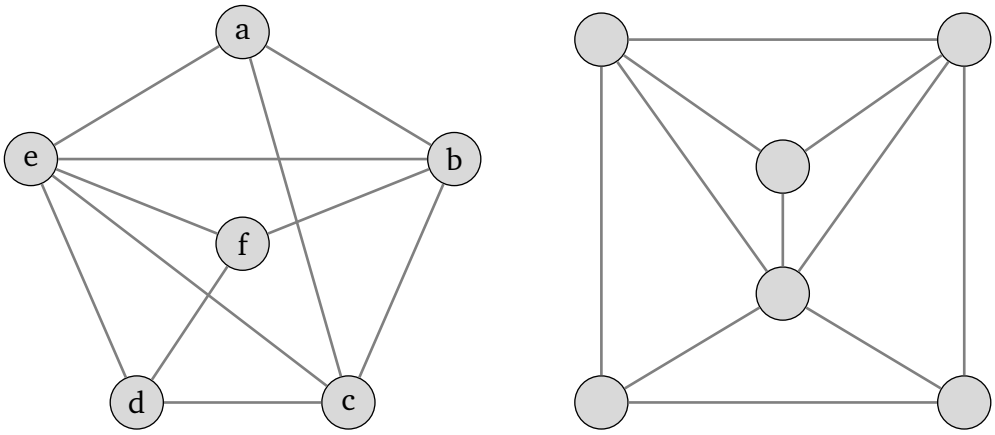
(c) The utilities graph $K_{2,5}$

(d) A connected graph with 3 vertices of degree 1, 2 vertices of degree 2, 1 vertex of degree 3, and 1 vertex of degree 4. Label the degree of each vertex on your drawing.

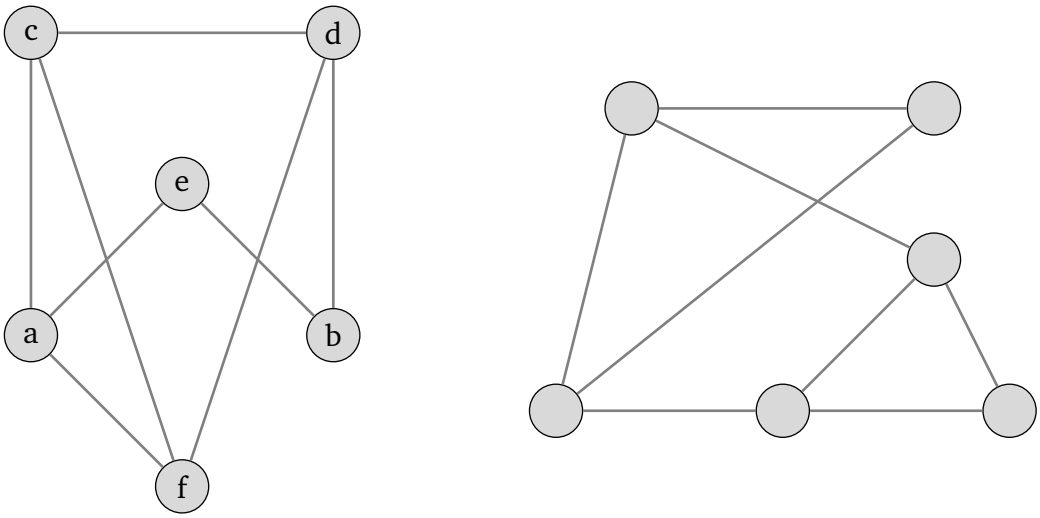
(e) A connected graph with 8 vertices and 7 edges, **no loops, and no multiple edges**.

4 (10 points)**.** For each pair of graphs, determine whether they are isomorphic or not. If yes, state the isomorphism by assigning letter names to the vertices in a way that makes the shown graphs equivalent. If no, explain why.

(a)

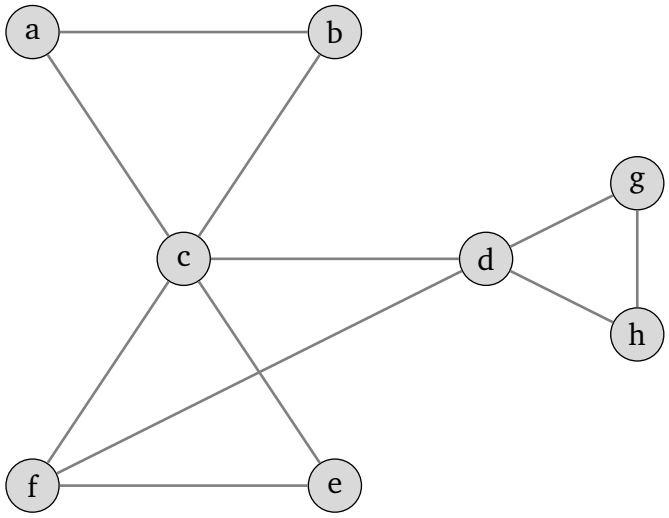


(b)

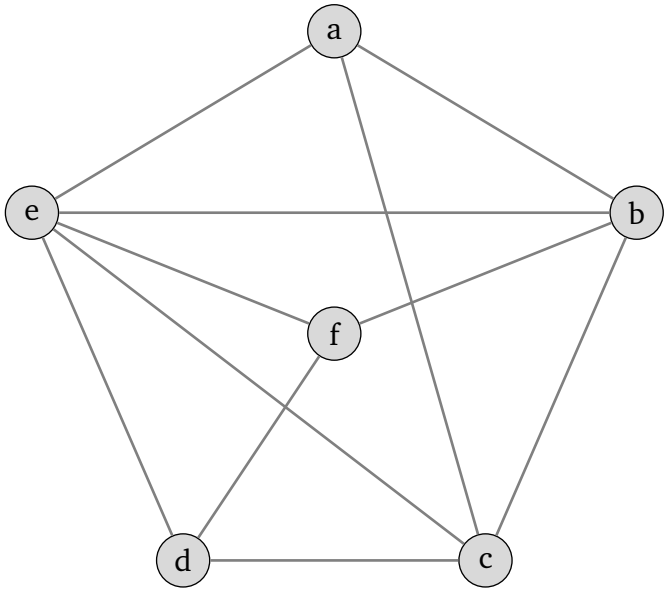


5 (10 points)**.** For each of the shown graphs, determine whether there is an Eulerian circuit or an Eulerian trail. If either one exists, draw it clearly, showing where it starts and ends. If neither a path nor a circuit exists, explain why.

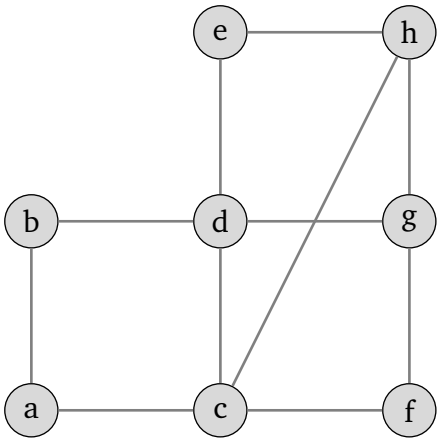
(a)



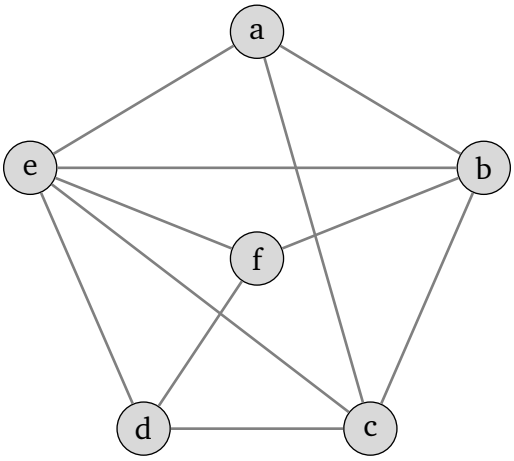
(b)



6 (5 points)**.** Determine whether there is an Eulerian circuit or an Eulerian trail. If either one exists, draw it clearly, showing where it starts and ends. If neither a path nor a circuit exists, explain why.



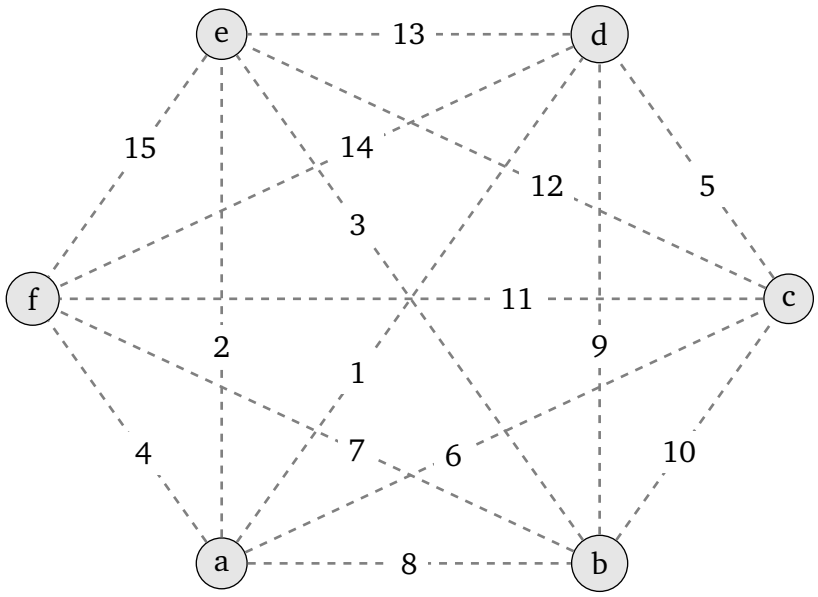
7 (5 points)**.** Show that the following graph is planar by finding an equivalent planar drawing.



8 (5 points)**.** Draw two distinct (non-isomorphic) non-planar graphs.

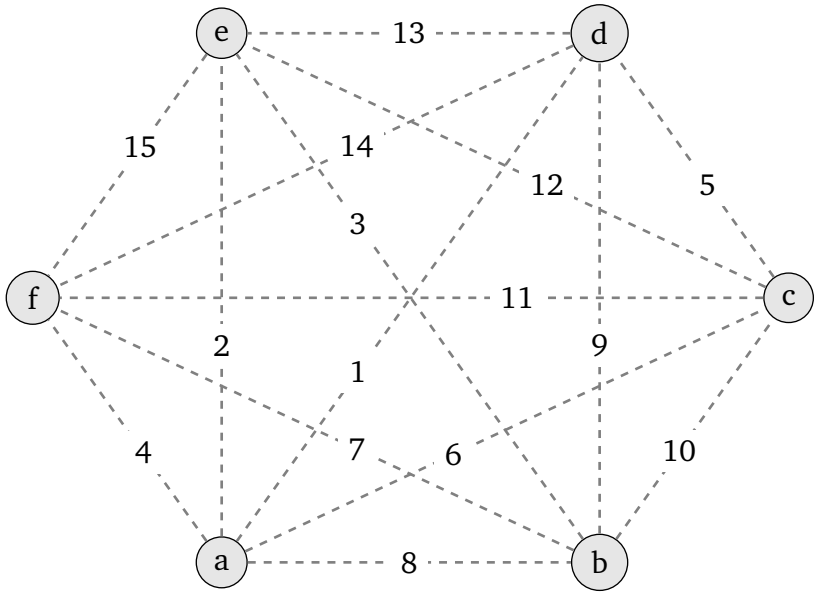
9 (10 points)**.** Use a specified algorithm to find a Hamiltonian cycle through the weighted graph and compute the weight of your cycle. Highlight the cycle on the graph.

(a) Use the Greedy algorithm starting at the vertex **a**.



Weight:

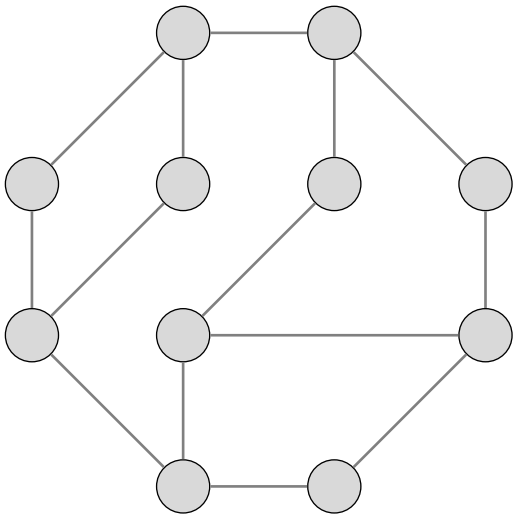
(b) Use the Edge-picking algorithm.



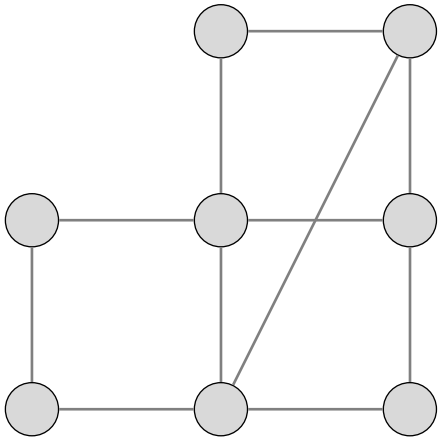
Weight:

10 (10 points)**.** For each of the following graphs either find a coloring using only 2 colors, or prove that more than 2 colors are needed by finding a cycle of odd length.

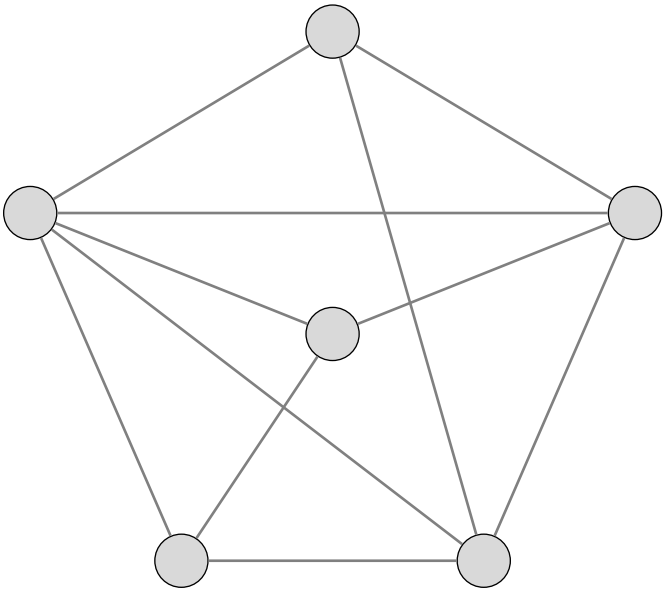
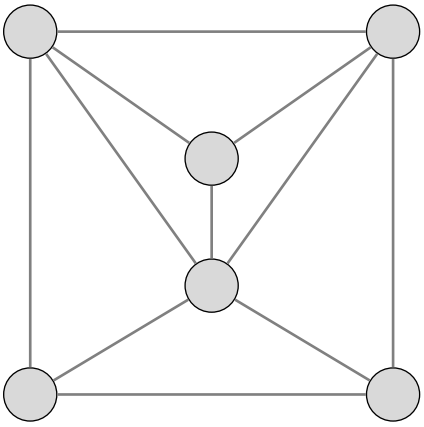
(a)



(b)



11 (5 points)**.** Find an optimal coloring for the given graph, using as few colors as possible. What is the chromatic number of this graph?



12 (5 points)**.** Draw a dual graph for the given graph; you may draw over the given graph, but must also **redraw your answer next to it**.