Practice Final

Math 400

Last Name	First Name	Date

This assignment is closed books, closed notes.

All your scratch work will be collected with the test and discarded.

All electronic devices besides 4-function calculators are prohibited.

Fully justify your answers and show all work in order to maximize your partial credit.

Leave your answers with simplified radicands and fractions in lowest terms.

Do not round anything unless directed.

1 (10 points)**.** Evaluate the integral, leave the answer in a simplified form, do not round.

$$\int_{0}^{2\pi/3} 7e^{3\sin(x)}\cos(x)\,dx =$$

2 (10 points)**.** Evaluate the indefinite integral, leave the answer in a simplified form, do not round.

$$\int x^{11}\sqrt[7]{9x^6+10}\,dx =$$

3 (10 points)**.** Evaluate the integral by interpreting it as an area under a curve, leave the answer in a simplified form, do not round.

$$\int_{-3}^{3} \left(8 + \sqrt{9 - x^2}\right)\, dx =$$

4 (10 points). Set up the right Riemann sum R_{10} to approximate the area under the curve of

$$f(x) = \cos(2x) + 4$$

from $a = 2$ to $b = 8$ using 10 rectangles and a regular partition.

$$\sum_{i=c}^{d} f(x_i)\Delta x, \text{ where}$$

- $\Delta x =$
- $x_i =$
- $f(x_i) =$
- $c =$
- $d =$

5 (5 points)**.** Determine the value of the limit conceptually, that is, without using technology or a table of values. Provide a detailed explanation.

$$\lim_{x \to 0^+} \ln\big(\tan(x)\big)$$

6 (10 points)**.** Evaluate the limit, leave the answer in exact form, do not round. Justify all steps with limit laws and/or appropriate theorems.

$$\lim_{x \to 0^+} \left(1 + \sin(6x)\right)^{2/x}$$

7 (5 points)**.** Evaluate the limit, leave the answer in exact form, do not round. Justify all steps with limit laws and/or appropriate theorems.

$$\lim_{x\to 2} \frac{\int_2^x (3t - 5e^{t-2})\, dt}{x-2}$$

8 (10 points). Find the value of the limit or prove that it does not exist. Justify all steps with limit laws and/or appropriate theorems.

$$\lim_{x \to -\infty} \frac{\cos^2(x)}{x^3 + 1}$$

9 (10 points)**.** Prove the statement using a rigorous ϵ, δ proof. Your work should be legible, and all your logic should be clear and justified.

$$\lim_{x \to -5} \left(\frac{3}{10} x - 8 \right) = -\frac{19}{2}$$

10 (10 points)**.** Prove the statement using a rigorous ϵ, M proof. Your work should be legible, and all your logic should be clear and justified.

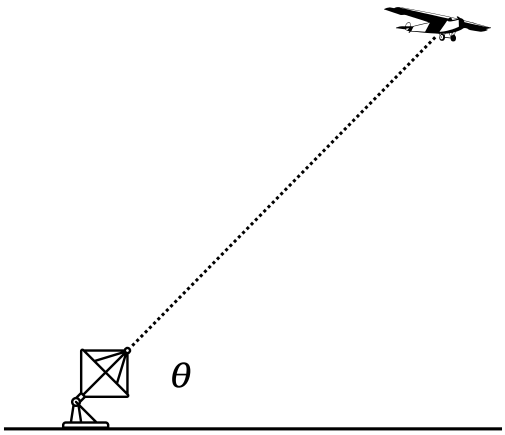
$$\lim_{x \to -\infty} \frac{8}{x^3} = 0$$

11 (10 points)**.** Find the equation of the tangent line to the curve

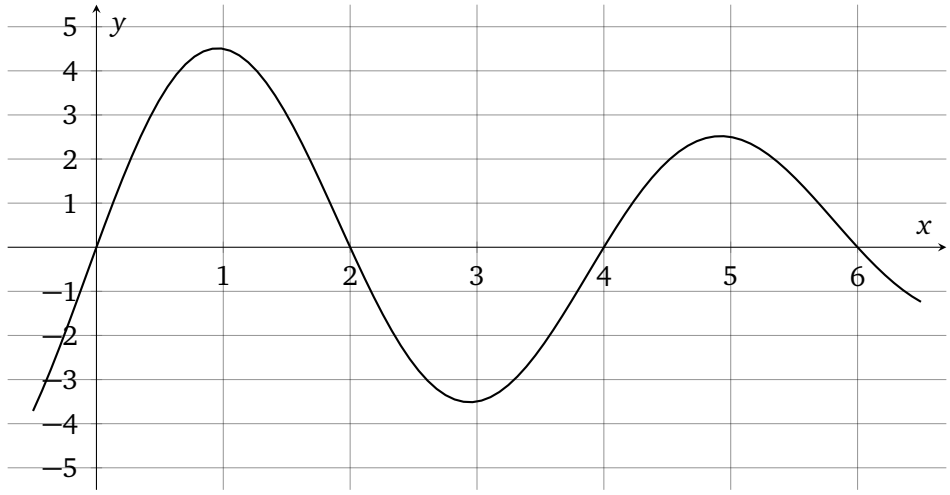
$$9\sec(xy) = \sqrt{x-6y}$$

at the point $(81, 0)$. Leave the answer in the slope-intercept form.

12 (10 points)**.** A plane flies horizontally at an altitude of 5 km and passes directly over a tracking telescope on the ground. When the angle of elevation θ is $\pi/3$, this angle is decreasing at a rate of $\pi/6$ radians per minute. How fast is the plane traveling at that time? You may round your answer to 4 significant digits.



13 (10 points). Let $A(x) = \int_0^x f(t)\,dt$, with $f(x)$ shown in the graph below.



Restrict your answers to values $0 \le x \le 6$. Explain your reasoning.

(a) At what x value(s) does $A(x)$ have a local max?

(b) At what x value(s) does $A(x)$ have a local min?

14 (5 points)**.** Use the limit definition of the derivative to find an expression for $f'(x)$, given that

$$f(x) = \sqrt{16 - x}$$

$f'(x) =$

15 (10 points)**.** Let $f(x)$ be an always negative function such that $f'(x) > 0$ for all real numbers x. Justify your answers and show all work.

(a) Let $g(x) = (f(x))^6$. For what values of x will $g(x)$ be decreasing?

(b) Let $h(x) = f\,(f(x))$. For what values of x will $h(x)$ be decreasing?

16 (5 points)**.** Find the derivative of the function:

$$I(x) = \int_{\sin(x)}^{\pi^2} w^2 \, dw$$

$I'(x) =$

17 (10 points)**.** Determine whether each statement is True or False **in general**. Circle the appropriate response.

(a) If a is a *critical point* of the function $f(x)$, then $f'(a) = 0$.

True **False**

(b) If $f(x) > 4$ for every $x > 100$, and $\lim_{x\to\infty} f(x)$ exists and is finite, then $\lim_{x\to\infty} f(x) \neq \pi$.

True **False**

(c) If $f(x)$ and $g(x)$ are *integrable* functions and $g(x) \neq 0$, then:

$$\int \frac{f(x)}{g(x)}\,dx = \frac{\int f(x)\,dx}{\int g(x)\,dx}$$

True **False**

(d) If $f''(a) = 0$, then $f(x)$ has an *inflection point* at $x = a$.

True **False**

(e) If the function $f(x)$ is *differentiable* at the point $x = a$, then $f(x)$ is *continuous* at the point $x = a$.

True **False**

TOTAL POINTS: 150