NORMAL RV HOMEWORK.

Let $X \sim N(\mu = 54, \sigma = 8)$.

1. Find $P(X > 56)$.

2. Find $P(X < 30)$.

3. Find the 80th percentile of X.

4. Find the value of X with z-score of -5.5.

Suppose that the mean length of a 2018 music track is 171 seconds, with the standard deviation of 34 seconds. A researcher takes a random sample of 7 popular music tracks produced in 2018 and measures the sample mean $\overline{Y}$.

5. Assuming that the Central Limit Theorem can be applied to this population and this sample size, what is the approximate distribution of $\overline{Y}$, the sample mean?

6. What are the chances that the sample mean is at most 140 seconds?

7. What is the 10th percentile of the sample mean?

8. What are the chances that the sample mean is between 150 and 160 seconds?

The mean body weight of an adult American is estimated to be 80.7 kg (Walpole 2005), with standard deviation 12 kg (Floud 1998). Take a simple random sample of 20 adult Americans and measure the mean weight in the sample $\overline{W}$.

9. Assuming that the Central Limit Theorem can be applied to this population and this sample size, what is the approximate distribution of $\overline{W}$?

10. How likely is the mean weight in the sample to be greater than 90 kg?

11. Find $P(80 < \overline{W} < 85)$

12. Suppose that a certain elevator can safely transport weights up to 1500 kg. What are the chances that a simple random sample of 20 people will overload the elevator?

According to census.gov, the percentage of U.S. families with their own children under 18 in the household was 40% in 2022. Suppose we draw a simple random sample of 350 U.S. families and let C be the number of families in the sample with children under 18 in the household.

13. Is the random variable C discrete or continuous?

14. Describe the distribution of the random variable C.

15. Find $P(C \leq 120)$.

16. Find μ_C, the expected number of families with children in the sample.

17. Find σ_C, the standard deviation for the number of children in the sample.

18. Use a normal approximation with continuity correction to estimate the probability $P(C \leq 120)$.

ANSWERS.

1. 0.4012937

2. 0.001349898

3. 60.73297

4. 10

5. $\overline{Y} \sim N(\mu = 171, \sigma = 12.85079)$

6. 0.007926058

7. 154.531 seconds

8. 0.1448902

9. $\overline{W} \sim N(\mu = 80.7, \sigma = 2.683282)$

10. 0.0002642246

11. 0.5483845

12. $P(W_1 + \ldots + W_{20} > 1500) = P(\overline{W} > 1500/20) = 0.983176$

13. Discrete

14. $C \sim \text{Binom}(n = 350, p = 0.4)$

15. 0.01607099

16. 140

17. 9.165151

18. 0.01668412