

COUNTING

TEXT:

LAST NAME	FIRST NAME	DATE
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1 (4 points). Compute the values of these expressions!

(a) $6!$

(c) $\frac{500!}{(500-2)!}$

(b) $(20-16)!$

(d) $\frac{7!}{2!(7-2)!}$

2 (1 point). Alice can choose to wear one pair of jeans out of three, one top out of four, and one scarf out of ten. How many different outfits can Alice create?

3 (1 point). Sean wants to borrow 6 of Morgan's books, but Morgan will only give him two books at a time. In how many ways can Sean choose which two books to borrow?

4 (1 point). Derek has 420 tracks in his Reggae collection. He wants to make a playlist without repetitions. How many distinct 3-track playlists can he make if the order of songs is important to him?

5 (1 point). ACME Space Corp needs to choose 3 astronauts out of the pool of 7 qualified candidates, where 5 of the candidates are pilots and the other 2 are scientists. If all choices are equally likely, what are the chances that all of the chosen individuals are pilots?

6 (4 points). A state license plate consists of 5 characters. Each character can be either an Arabic digit or a capital English letter (so there are 10 digits and 26 letters). State your answers as decimals.

(a) Find the number of distinct license plates.

(b) What are the chances a random license plate starts with the word CAT?

(c) What are the chances a random license plate does not contain the letter Z?

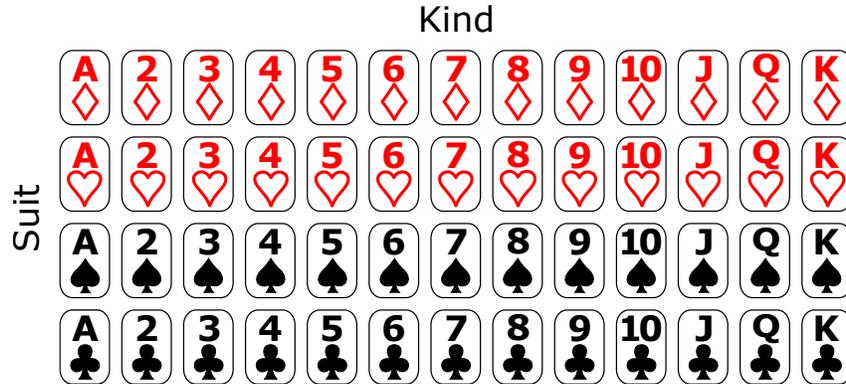
(d) What are the chances a random license plate does not contain the letter Z, given that it consists entirely of letters?

7 (2 points). A basket contains 50 green apples and 40 red apples. Shawneese picks 10 apples without looking.

(a) What is the probability Shawneese picks 10 green apples?

(b) What is the probability Shawneese picks 5 green and 5 red apples?

8 (1 point). Harry sets a random 4-digit PIN on his bank account. If all PINs are equally likely, what are the chances that Harry's PIN repeats a digit at least once?



9. In the game of Five Card Draw each player gets five cards out of the standard 52-card deck. If the cards are well shuffled, then we can assume that every possible arrangement of five distinct cards (called **hand**) is equally likely.

- (a) How many different hands are there?

- (b) What are the chances of a *royal flush*: TJQKA, all of the same suit?

- (c) How likely is one to draw at least one Ace?

- (d) Find the probability of a *full house*: a three of a kind and a pair, for example QQQAA.

- (e) Find the likelihood of a *two pair*: two different pairs and one other card, like JJ773.

- (f) Find the probability of a *straight*, which is five cards in a sequence, where an Ace can be the lowest or the highest card, for example A2345, or 89TJQ, or TJQKA, with any combination of suits.

10. Penelope, Qin, Rina, Samira, and Tenzin are all interested in going to a new movie together. Chances are, not all of them can make it work with their schedules, so suppose that any subset of the five friends is equally likely to show up for the movie.

(a) What is the size of the sampling space?

(b) What are the chances that Penelope shows up, but Tenzin does not?

11. Suppose that an unknown password consists of 6 capital English letters, and all possible passwords are equally likely.

(a) What is the probability that the password contains a substring “MEW”?

(b) What is the probability that the password contains a substring “HEH”?