CENTRAL LIMIT THEOREM

TEXT:

LAST NAME	FIRST NAME	DATE

1 (5 points). IRS reports that the mean federal income tax paid in the year 2010 was \$8040, with the standard deviation of \$5000. IRS takes a random sample of 1000 tax payments.

(a) What is the approximate distribution of the sample mean?

(b) What is the probability that the sample mean is above \$8000?

(c) What is the probability that the sample mean is between \$7700 and \$7900?

(d) What is the probability that the sample mean is less than \$7950?

(e) Find the 99th percentile of the sample mean.

2 (5 points). Suppose that the weight of an adult milk cow is known to have the mean of 462 kg and the standard deviation of 29 kg, but the exact distribution of the weight is unknown. When moving from farm to farm, cows are transported on a truck that can safely carry the load of up to 5000 kg.

(a) What is the probability that 1 cow weighs more than 500 kg?

(b) Suppose we load 10 random cows onto the truck. What is the approximate distribution of the sample mean weight $\overline{X}$?

(c) Find the probability that 10 cows weigh more than 5000 kg in total. [*Hint: this is the same as the probability that the mean weight in the sample exceeds* 500 *kg.*]

(d) Now suppose we load 11 random cows onto the truck. What is the mean weight in the sample, if the total weight of 11 cows is 5000 kg?

(e) What are the chances that 11 cows together weigh more than 5000 kg in total?

3 (3 points)**.** A random sample of 120 lady bugs is chosen out of the population where 38% of individuals have dark spots, while the rest are plain red.

(a) What is the distribution of the random variable X, which counts the number of spotted lady bugs in the sample?

(b) What is the probability that at least 42 lady bugs in the sample are spotted?

(c) Use a normal approximation (with continuity correction) of the distribution above to compute the same probability.

4 (7 points). Studies suggest that about 88% of the world human population is right-handed. Take a random sample of size $n = 180$ and let X be the number of right-handed individuals in the sample.

(a) What is the distribution of X?

(b) Find the expected number of right-handed individuals in the sample.

(c) Find the standard deviation for the number of right-handed individuals in the sample.

(d) Find the probability that at most 150 individuals in the sample are right-handed.

(e) Find the sample proportion $\hat{p}$ for a sample of size 180, which has 150 right-handed individuals.

(f) What is the approximate distribution of the sample proportion $\hat{P}$?

(g) Use the CLT to approximate the likelihood of the sample proportion being at most $\hat{p}$, and compare the result with the exact binomial probability in part (d).